Official Sovereign Debt*

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Abstract

This paper studies official sovereign debt empirically and theoretically. Official sovereign debt is more than half of the total sovereign debt in emerging markets and tends to rise during default episodes, much more than debt with private creditors. We develop a model with official and private debt where the sovereign can partially default on each of its debts. A fraction of the defaulted debt accumulates during a default episode, which resolves when the sovereign pays back its accrued obligations. We find that official debt is longer-term than private debt and more concessional. These differences across debts allow our model to rationalize the stylized facts of emerging markets. Counterfactual analysis suggests that official debt is welfare improving for indebted economies and finds the possibility of voluntary swaps of private for official debt during debt crises.

*The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. Contact information: arellano.cristina@gmail.com; barre843@umn.edu

1 Introduction

A large portion of the sovereign debt in emerging markets is with official lenders including bilateral loans with other sovereign governments and loans with multinational organizations. The recent empirical work of Horn, Reinhart, and Trebesch (2020) emphasizes the importance of official lending for many countries historically and its role in coping with adverse shocks. Theoretical work on sovereign debt, however, has mainly focused on debt and default problems in contracts with private creditors. This paper provides an integrated framework with official and private debts and explores the role of official debt in being the asset of choice during episodes of financial distress.

We start by analyzing a panel dataset covering 50 years and 30 emerging markets that contains debt series and their decomposition across official and private creditors. We build on the accounting framework of Arellano, Mateos-Planas, and Ríos-Rull (2023) to classify partial defaults and default episodes. We find that official sovereign debt corresponds to more than half of the total external sovereign debt for these countries and that the duration of this debt is longer than for private debt. We then document the patterns of debt during default episodes and downturns. We find that official debt tends to grow during default episodes and accounts for much of the dynamics of total debt during these events. Private debt, in contrast, remains pretty stable during these events.

We then develop a sovereign default model with official and private debt. Our framework consists of a sovereign government in a small open economy that faces a stochastic stream of income and can choose to partially default on its debt payments, selectively on official or private debt. Official debt differs from private debt in that it is of longer maturity and more concessional, with lower recovery factors during defaults. Upon a partial default, the defaulted debt accumulates: when the sovereign partially defaults on coupons of official or private debt, a fraction of that amount, which depends on a recovery factor parameter that is debt-specific, is added to the total debt due next period. Partial default is costly because it induces resource costs that depend on the intensity of the default, yet it is a flexible policy as the government decides on the duration and intensity of default, separately for each type of debt. The government raises funds by borrowing from official lenders and private lenders at interest rates that compensate lenders for potential default losses. Borrowing with these two assets is always possible, even during default episodes.

We derive an optimality condition for partial default and show that partial default incentives are bigger when the values of official and private debt are large as well as when bond prices on new borrowings are low. Bond prices matter for partial default because they encode the value of the accumulated defaulted coupons; a low price lowers the value of those claims and increases the benefit to the sovereign for defaulting today because less will be repaid in expected value.

An important aspect of our framework is that partial default is a period-by-period decision by the sovereign and borrowing is permitted during default. The sovereign with official long-term debt may choose to default on this period's coupons, experience a cost, but the following period may choose to repay the coupon and eliminate any cost. Moreover, during a default, it may borrow from lenders. These features are different from traditional sovereign default theory, as in Chatterjee and Eyigungor (2012), where default is a permanent decision that eliminates current and future obligations, precludes any borrowing, and leads to permanent costs, such as financial autarky. Our environment is more consistent with the evidence in emerging markets where debt grows during defaults and we find it matters for the debt sustainability of different debts.

We characterize theoretically how the differences between private and official contracts map into debt capacity and find that official debt gives greater debt capacity to the sovereign. To show these results, we use a simplified model economy with no shocks and zero recovery factors upon a partial default. We show that official debt contracts can expand the budget set of the sovereign more than private debt. The higher debt capacity of official debt arises because it is a long-term duration asset and partial default is a period-by-period decision. Official contracts can effectively constrain future governments from borrowing, as any pledgeable resources, namely the default cost, are committed to the long-term official contract. Private, short-term, contracts do not constrain future governments and therefore only the one-period ahead pledgeable resources can be used for them, as the sovereign can turn around after default and borrow again fresh loans. We also relate to traditional sovereign default theory, and show that the debt capacity of both assets would be the same under the common assumption of full default, no borrowing during defaults, and permanent costs.

We perform a quantitative evaluation of the model and map it to emerging market data. We use unconditional moments in the parameterization of the model and show that the model can reproduce salient patterns during default episodes. We target first and second moments of official and private debt, partial default, and the ratios of debt service to debt for official and private debt. Our moment-matching process recovers parameters for the duration of debts, the recovery factors, and default costs. Importantly, our exercise implies that official debt is of longer duration and more concessional, two properties that are consistent with external evidence.

Our baseline model reproduces an economy with debt ratios as in the data, official debt is about 2/3 of the total debt. Moreover, we show that as in the data, in the model debt grows during defaults, more so for official debt. The magnitudes of these increases are similar to those in the data.

We use the baseline model to perform counterfactuals. We first evaluate the role of official debt by comparing our baseline to an economy with only private debt. This exercise reveals that official debt helps in reducing the volatility of consumption and the level of private spreads. Moreover, official debt is welfare improving for indebted economies. Second, we study the design of official contracts by considering economies where official debt is of shorter duration and less concessional. These are motivated by the liquidity programs from the International Monetary Fund and the Federal Reserve, which offer short-term loans with little default risk to various countries. We find that the more traditional long-term official contracts tend to be better for welfare. Third, we study the feasibility of voluntary swaps of debts. We find room for voluntary swaps of private for official debt, especially during debt crises that start with sizable private debt. Swaps can be Pareto improving as they increase the value of debts and the welfare of sovereigns.

Literature Review Our work contributes to the emerging literature studying official lending. In recent work, Horn, Reinhart, and Trebesch (2020) documents how extensive official lending has been historically for many countries around the world. They make a compelling case that an important role for official debt is coping with the economic consequences of disasters, including natural and financial adverse shocks. Schlegl, Trebesch, and Wright (2019) studies the seniority of official and private debt. Using measures of debt in arrears and haircuts from default episodes, they argue that official lenders, especially bilateral ones, are more junior than private debt creditors. The rising role of official Chinese lending to emerging markets since the early 2000s, as documented by Horn, Reinhart, and Trebesch (2021), has also sparked some work focusing on official Chinese loans. Our work complements these empirical findings, by using a comprehensive dataset on official and private debt and arrears, and taking an aggregated view of official debt. Our work illustrates a novel property, namely that official debt is used more heavily during default episodes.

Some theoretical work on official lending has studied multinational lending, from institutions like the International Monetary Fund, and has taken the view that these loans are not defaultable. Boz (2011) and Kirsch and Rühmkorf (2017) enrich a sovereign default model with multinational loans that are not defaultable and show that the lack of default risk from multinational lending makes it useful at providing insurance. Our work on aggregated official lending finds a similar insurance role for this debt, but unlike previous work and consistent with the evidence allows for this debt to also be defaultable.

A main property of official lending is its longer maturity. As such, our work contributes to the large literature that has analyzed the interactions between default risk and the maturity of sovereign debt. Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012) show that the inclusion of long-term bonds improves the quantitative performance of sovereign debt models, and examine the so-called "debt dilution problem". Arellano and Ramanarayanan (2012) document that governments in four emerging markets shorten the maturity of debt issuances during sovereign crises, and show in a calibrated model that long-term debt provides a better hedge against future fluctuations in spreads, while short-term debt provides more incentives to repay. Perez (2017) extends the sample to a panel of 34 countries and finds a similar pattern. Niepelt (2014) analyzes how sovereign risk affects the maturity structure and rollover decisions of public debt by subsequent policymakers. Hatchondo, Martinez, and Sosa-Padilla (2016) study the effects of debt dilution on sovereign default risk and show that eliminating dilution increases the optimal duration of sovereign debt by almost 2 years. In related work, Sánchez, Sapriza, and Yurdagul (2018) document that sovereign debt maturity and duration generally exceed a year and move procyclically, and highlight that output volatility, impatience, risk aversion, and sudden stops are key determinants of maturity.

More recently, Dovis (2019) explores an optimal risk-sharing arrangement between the government and foreign lenders in a model with informational and commitment frictions and finds that the constrained efficient allocation can be implemented with non-contingent defaultable bonds and active maturity management. Cole and Kehoe (2000) built a sovereign debt model with self-fulfilling debt crises, and point out some potential disadvantages of short-term borrowing. Within this framework, Bocola and Dovis (2019) introduce risk-averse lenders and endogenous debt maturity and find that tracking the evolution of debt maturity provides information on the relative importance of fundamental and nonfundamental risk during the eurozone crisis. Aguiar et al. (2019) explore the interaction between sovereign default and maturity choice, finding that the sovereign should actively manage only short-term debt while remaining passive in long-term bond markets. Mihalache (2020) argues that sovereign default episodes are resolved by restructuring the debt through renegotiations, finding that debt relief is implemented by extending the maturity of the debt, as opposed to changing its face value. Bigio, Nuño, and Passadore (2023) characterize an optimal debt-maturity management problem in a model with finite-maturity bonds of different maturities that rationalizes Spain's debt management during the sovereign debt crisis.

We re-consider these interactions in the framework of rich default episodes and partial default of Arellano, Mateos-Planas, and Ríos-Rull (2023) and recover new insights. We find that the usefulness of the insurance properties of long-term debt, discussed in Arellano and Ramanarayanan (2012), are magnified in this framework and overturns the conventional welfare ranking of Chatterjee and Eyigungor (2012) that short-term debt is preferred. In our framework, official long-term debt welfare dominates short-term debt because it carries higher debt capacity.

2 Empirical Properties of Official and Private Debt

In this section, we document some properties of official and private lending and sovereign partial defaults using 50 years of data from emerging markets. We will extend the analysis in Arellano, Mateos-Planas, and Ríos-Rull (2023) (thereafter AMR) by focusing on differential patterns of official and private debt using their accounting framework to organize the data.

2.1 Accounting

To fix ideas for our work, we will revisit the accounting framework of AMR and apply it to our case of analyzing disaggregated private and official debt.

Flow Financial Variables. Each period, the sovereign owes official lenders an amount $\tilde{f}_t$ and private lenders an amount $\tilde{b}_t$, which are the sum of all the coupons from past issuances due at t. As we will see later, these amounts include not only the promised coupons at t from newly issued bonds in previous periods but also the current obligations that result from past partial defaults. We consider a flexible partial default policy that is applied to the payment dues, given by d_t^f and d_t^b for official and private debt respectively. Defaults $\{d_t^f, d_t^b\}$ imply that the sovereign pays in period t the amount $(1-d_t^f)\tilde{f}_t$ to official lenders and $(1-d_t^b)\tilde{b}_t$ to private lenders, and does not pay $d_t^f\tilde{f}_t$ and $d_t^b\tilde{b}_t$. Given the default policies, debt service for official and private debt are $(1-d_t^f)\tilde{f}_t$ and $(1-d_t^b)\tilde{b}_t$, debt service for total debt is the sum of these two debt services. Partial default for each type of debt is defined as the fraction of the debt due defaulted on and therefore equals d_t^f and d_t^b for official and private debt. Partial default for total debt d_t in turn equals, $(d_t^b\tilde{b}_t + d_t^f\tilde{f}_t)/(\tilde{b}_t + \tilde{f}_t)$.

Long-Term Bonds with Partial Default. We map the data into a tractable structure for long-term debt contracts that consist of perpetuity bonds with coupon payments that decay, as in Hatchondo and Martinez (2009). We allow for different durations for each type of debt by considering different decay parameters for the bonds, ϑ^f and ϑ^b for official and private debts respectively. For each type of debt $a = \{f, d\}$, a borrowing contract specifies a price q_t^a and a value ℓ_t^a such that the sovereign receives $q_t^a\ell_t^a$ units in period t and promises to pay, conditional on not defaulting, $(r+\vartheta^a)(1-\vartheta^a)^{n-1}\ell_t^a$ units in every future period $t+n$ for $n = 1, 2, ..., \infty$. The constant factor $(r+\vartheta^a)$ is a normalization that has no bearing on the analysis, it simply makes the risk-free bond price 1 with r being the international risk-free rate. This normalization only changes the units of the debt due, such that $\tilde{a}_t = (r+\delta_a)a_t$. These contracts are tractable because they encode a rich structure of debt issuances into a single state variable for each type of debt, namely f_t and b_t, with their laws of motion. A sovereign that in period t pays in full its debts due $(r+\vartheta^f)f_t$ and $(r+\vartheta^b)b_t$, and borrows $\ell_{f,t}$ and $\ell_{b,t}$ will have in period $t+1$ states equal to $f_{t+1} = (1-\vartheta^f)f_t + \ell_{f,t}$ and $b_{t+1} = (1-\vartheta^b)b_t + \ell_{b,t}$, for official and private debt respectively. These states include the coupons from the legacy debt and the new borrowing.

We assume that partial default of intensity d_t^a for each type of debt a, reduces the debt service and can trigger defaults on all future coupons, encoded in the legacy debt $(1-\vartheta^a)a_t$, of intensity $\mu^a d_t$. We interpret the parameter μ^a as arising from default acceleration clauses, which are common in private bonds, and also to reflect that in practice many restructurings exchange streams of bonds with payoffs arising in the future. As in AMR, we assume that the defaulted coupons and defaults on legacy debt, namely $(r+\vartheta^a)d_t a_t + (1-\vartheta^a)\mu^a d_t a_t$, result in new obligations, $\kappa^a d_t a_t$, that are due in the future. The factor κ^a is a parameter that captures the empirical observation that during default episodes, sovereigns accumulate their defaulted debt and, in some cases, restructure their obligations with their creditors. Contracts with lower κ^a are more concessional: defaults on those contracts result in higher discharge of debt.

The law of motion of the states, therefore, incorporates the legacy debt, the accumulation of defaulted coupons, and new borrowing:

$$a_{t+1} = (1-\vartheta^a)(1-\mu^a d_t^a)a_t + \kappa^a \; d_t \; a_t + \ell_{a,t}. \tag{1}$$

Note that a partial default $d_t > 0$ does not necessarily reduce debt a_{t+1} relative to a_t. Debt can actually increase when the recovery factor is sufficiently high because the defaulted coupons are accumulated with interest. Also, debt can increase if borrowing $\ell_{a,t}$ is positive. As we document below, we find differential patterns of private and official debt during partial defaults. Differences in the availability of credit during private defaults across debt classes as well as distinct recovery factors will determine these patterns.

Debt, Duration, Spreads, and Default Episodes. We measure the debt levels and their duration for each class of debt using streams of contractual payments due. At time t, for each class of debt, the contractual payments due are the promises to pay in period $t+j$, conditional on not defaulting, $\tilde{a}_t^{t+j}$ for $j = 1, 2, ..., \infty$. These promised payments $\tilde{a}_t^{t+j}$ are deterministic sequences and, in general, will differ from the actual payments due, which are stochastic at time t because of partial default. We define the level of private and official debt at t as the present value of the contractual payments due and the duration of these debts as the corresponding "Macaulay duration" with flows discounted at the risk-free gross interest rate $(1+r)$. Our bond structure implies that a sovereign with end-of-the-period state a_{t+1} has a sequence of contractual promises $(r+\vartheta^a)(1-\vartheta^a)^{j-1}\ a_{t+1}$ for $j = 1, 2, ..., \infty$, and a debt level of a_{t+1} with associated duration of $\frac{1+r}{r+\vartheta^a}$.

In practice, because of default risk, the sovereign's promises to pay the stream $\{\tilde{a}_t^{t+1}, \tilde{a}_t^{t+2}, ..., \tilde{a}_t^{\infty}\}$ will have a market value different from the debt level defined above. As is standard, we can use the market value of the debt and the streams of contractual payments to define the yield-to-maturity, which is the constant discount rate that equates these two. The sovereign spread s_t is the difference between the yield-to-maturity and the risk-free rate. For our perpetuity contracts, the market value of debt of class a is $q_{a,t}(r+\vartheta^a)a_{t+1}$ as future defaults are applied uniformly across all these securities of class a. The sovereign spread is inversely related to q_t and equals $s_t = (r+\vartheta^a)(\frac{1}{q_t} - 1)$.

We flag a default episode as a sequence of periods with consecutive positive partial defaults and define its length by the number of such periods. An episode of length $N+1$, which starts in period t and ends in period $t+N$, has $d_{t+j} > 0$ for $j = 0, 1, ..., N$ and $d_{t-1} = d_{t+N+1} = 0$. The sequences of official and private debt level, debt service, as well as the sequence of partial default for official, private, and total, in the default episode, are given by $\{a_{t+j}, (r+\delta_a)(1-d_{t+j})a_{t+j}, d^a_{t+j}, d_{t+j}\}$ for $a = \{f, b\}$ and $j = 0, 1, ..., N$.

2.2 Empirical Findings

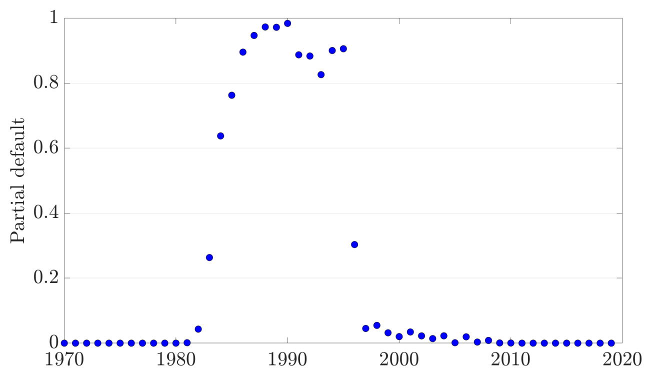
We use the debt statistics from the World Development Indicators (WDI), International Debt Statistics (IDS), and the Debtor Reporting System, all from the World Bank, to empirically measure the variables of interest in our accounting framework at an annual frequency. From these data, we use the debt obligations for the government, defined as public and publicly guaranteed (PPG), for both flow and stock variables. We focus on the total debt obligations, as well as the decomposition across these obligations between official and private credit. Debt obligations with private creditors include debt in the form of bonds and loans, and debt with official creditors includes loans with bilateral governments, multinational organizations, and trade credit. We also collect data on Gross Domestic Product in constant dollars which we log and linearly detrend. We also use government EMBI+ spreads from the Global Financial Database. The dataset is annual and corresponds to a panel of 30 emerging countries from

1970 to 2019.

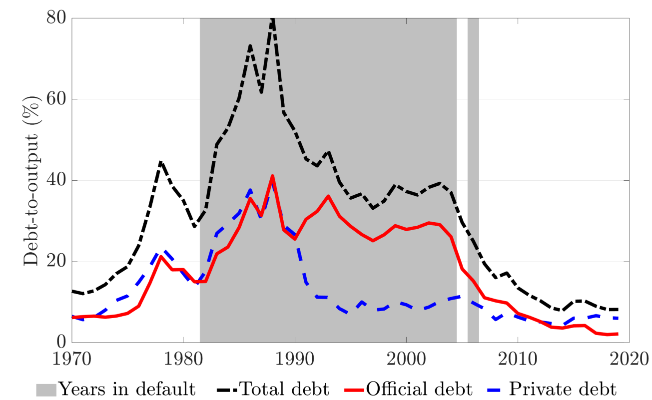
We document patterns of the variables of interest discussed in our accounting framework by mapping the dataset to our accounting. We analyze properties of official and private debt, in terms of levels, volatilities, and their comovement with partial default and output.

2.2.1 Case Studies

Figure 1: Peru: Partial Default, Official, Private, and Total Debt

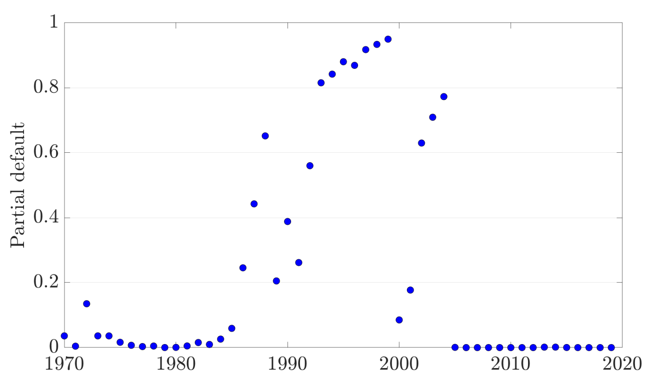


(a) Partial Default

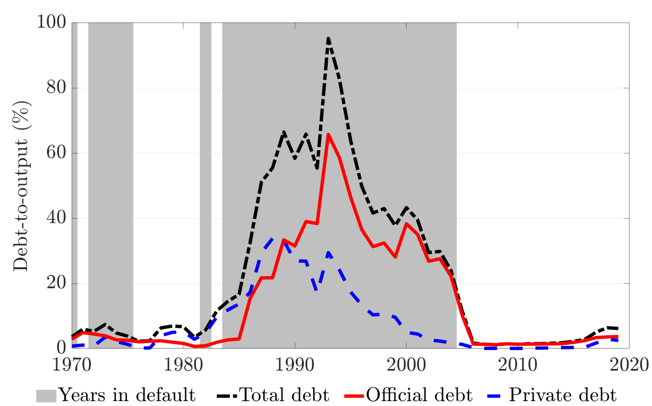


(b) Debts

Figure 2: Nigeria: Partial Default, Official, Private, and Total Debt



(a) Partial Default



(b) Debts

We start by describing the times series properties of partial default and total, official, and private government debt for two countries, Peru and Nigeria. These countries feature time series patterns that are similar to the average patterns across countries. Figure 1 plots the times series for Peru; the left panel in the figure has the times series for partial default, and the right panel contains the time series for the debts. The left panel shows that Peru had a long default event, with partial default increasing from the early 1980s from 0 to about 1 in 1990. Partial default fell after that but remained positive well into the early 2000s. The right panel plots the three debt series, the black dotted line corresponds to the total debt to output, the red solid line is the official debt to output and the blue dashed line is the private debt to output. The sum of official debt and private debt is the total debt. The figure also contains shaded bars, that correspond to the periods of positive partial default. Before the default episode started, the level of official and private debt in Peru was similar and equal to about 15% of output. When the default episode starts, both debts grow and reach close to 40% in the late 1980s. The official debt remains elevated until the end of the episode at about 30% of output. The private debt in contrast falls during the episode and remains at about 10% of output for much of the latter part of the episode. The total debt at the end of the default episode is similar to before the beginning of the episode, but this end level is largely composed of official debt, in contrast to the beginning, where the shares of official and private were very similar.

Figure 2 plots the times series for Nigeria. The structure of the figure is the same as for Peru. Nigeria experienced four default episodes according to our accounting. We will focus on the long episode that starts in the mid-1980s and runs through the mid-2000s. The left panel shows that partial default starts small and increases during the episode reaching its peak in the mid-1990s. Early 2000s partial default is minor but in the mid-2000s it increases again. The right panel shows that the patterns of debts in Nigeria were similar to those in Peru with some differences. Right before the default episode starts, Nigeria has mainly private government debt, about 15% of output. The beginning of the default episode features a rise in both official and private debt. By the late 1980s, private debt stops rising but official debt rises to about 60% of output. At the end of the default episode, Nigerian debt is mainly official, at about 25% of output.

These examples illustrate that the dynamics of official debt are crucial in understanding the evolution of debt during default episodes. Official debt is a major source of financing during default episodes, and in some cases substitutes the use of private debt. Moreover, the increase in total debt is driven mainly by official debt.

2.2.2 Descriptive Statistics

We now compile some descriptive statistics from the panel data. Table 1 reports the first and second moments for debt levels, debt service for total, official, and private, all relative to output, as well as for partial default. The mean total debt to output in the panel data is 32%. The share of official debt is 63% and the share of private debt is 37%; official debt to output is 20% on average and private debt to output is 13%. Total debt service is 3.6% of output, about 50% of this debt service is paid to official lenders and half of it to private lenders.

An interesting feature is that although official debt is about 50% higher than private debt, the debt service of official debt is smaller than that for private debt. As we explore further in the calibration of our quantitative model, these patterns imply that official debt is of longer duration. Note that in our accounting framework, the ratio of debt service to debt, conditional on zero partial default, is equal to $r + \vartheta^a$. These statistics imply that $\vartheta^b > \vartheta^f$, which implies that official debt is of longer duration.

The second column in Table 1 reports the standard deviations of the debts. The standard deviations reported are the average ones across the countries in the sample. The volatility of the debt is high, with comparable coefficients of variation across official and private debt. The volatility of debt service is 2% and about half of that for official and private.

Table 1 also reports the correlations of debt with partial default and output. We report the mean time series correlations of the variables across countries. The official and private debt levels are positively correlated with partial default and negatively correlated with output. The magnitude of the correlations, however, is stronger for official debt. The correlations of debt service depend on the type of debt. Official debt service is positively correlated with partial default and negatively correlated with output. Private debt service, in contrast, is negatively correlated with partial default and positively correlated with output, although the magnitude of these correlations is small. These co-movements imply that periods of high partial default and low output are associated with high official debt and debt service.

The last row of the table reports the statistics for the variable partial default. The frequency of positive partial default in the sample is 40%, the mean of partial default conditional on being positive is 32% and the mean standard deviation of partial default conditional on being positive is 24%.

Table 1: Official, Private, and Total Debt

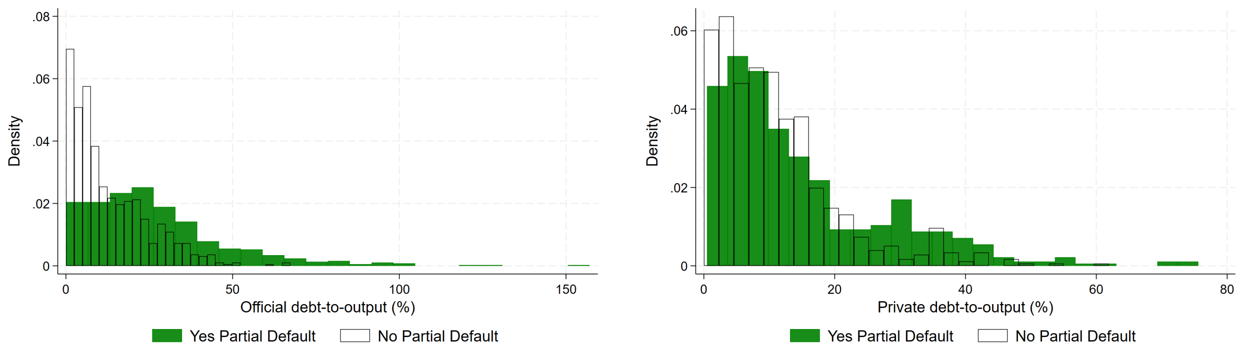
	Mean	Std Dev	Correlations with	
			partial default	output
Debt to output (in %)				
Total	32	18	33	-22
Official	20	12	38	-28
Private	13	8	16	-6
Debt service to output (in %)				
Total	3.6	2.0	1	-6
Official	1.6	1.0	16	-18
Private	1.9	1.6	-10	10
Partial default | partial default >0 (in %)	32	24		-2

Notes: Data is from the World Bank Databases and measured as public and publicly guaranteed (PPG). We obtain measures of debt levels, debt service, and partial default. Total corresponds to the total external PPG series. Official is the sum of debt from bilateral, multilateral, and trade creditors. Private is the sum of debt for bonds and loans with private creditors. Partial default is the ratio of total debt in arrears over the sum of total debt service and debt in arrears. Output is Gross Domestic Product in constant prices, log a linearly detrended. The standard deviations and correlations from emerging market data are means across countries of the statistics using country time series data.

Official Debt and Default We now analyze in more detail the patterns of official and private debt. We first assess how official and private debts vary with partial default. Figure 3 plots the histograms of official and private debt to output in the panel dataset across periods with no partial default and positive partial default. The white histograms are those with no partial default, while the green histograms correspond to the periods with positive partial default. The histograms show that both distributions shift to the right with partial default, namely official and private debt to output tends to increase uniformly with partial default. The shift, however, is much more pronounced for official debt, Many countries feature very small levels of official debt, less than 10% when they do not have partial default and this mass disappears with partial default, as most countries expand substantially their official debt holding. These histograms illustrate that for most emerging countries, partial default is associated with inflows of official loans, as the narratives above for Peru and Nigeria illustrated.

Table 2 reports the means of the variables of interest across states with no partial default and positive partial default. Debt to output is about 19% higher when partial default is positive

Figure 3: Distributions of Official and Private Debt



(a) Official

(b) Private

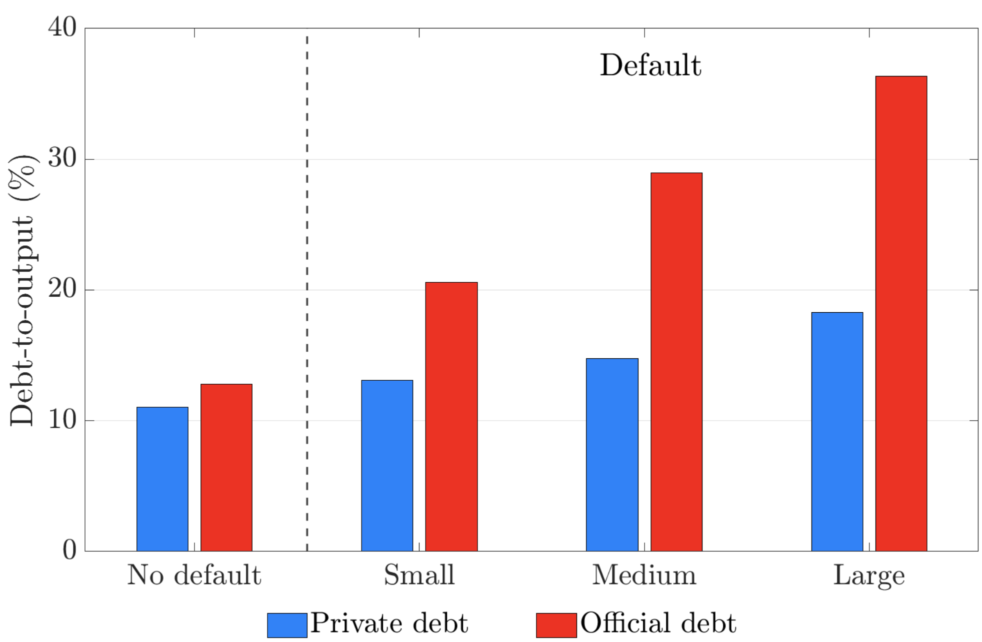
(44% vs 23%). The majority of this increase is due to an increase in official debt. Official debt to output increases by 16% while private debt to output increases only by 4%. Debt service to output increases on average with partial default—although the government is not paying all of its debt due, the higher debt implies that the government is paying more in terms of output for servicing the non-defaulted portion of the coupons. We do find that in periods with elevated partial default, above the 75 percentile of the partial default distribution, debt service on the total debt falls 2.6% of output. Official debt service increases by more than private debt service because of the additional inflows of official debt during these times. The table also illustrates that periods with positive partial default are associated with higher sovereign spreads and lower output; spreads are about 7% higher with partial default and output is 5% lower.

We now further decompose the patterns of debt across partial default. In Figure 4, we report official and private debts across finer partial default bins. We consider four bins for partial default. The bin "No default" contains the observations with no partial default. The bin "Small" contains observations with positive partial default but below the 25 percentile. The bin "Medium" contains observations with partial default between the 25 and 75 percentile. The bin "Large" contains the observations with partial default above the 75 percentile. The bars are the means of official and private debt to output across bins; the red bars correspond to official debt and the blue bars correspond to private debt. The figure illustrates that both debts increase with partial default and that the increase is sharper for official debt. Official debt is about 25 percentage points higher when partial default is in the top quartile (bin 4) relative to when partial default is zero (bin 0). Private debt in contrast is about 8% higher when partial default is in the top quartile relative to when it is zero.

Table 2: Default Flag: Total, Official, and Private Debt

	No default	Partial default > 0
Partial default	0	32
Debt to output (in %)		
Total	23	44
Official	13	29
Private	11	15
Debt service to output (in %)		
Total	3.1	4.1
Official	1.2	2.1
Private	1.8	2.0
Spreads	4	11
Output	2	-3

Figure 4: Official and Private Debt across Partial Default Bins



We now study the properties of default episodes for the 30 emerging markets, by analyzing dynamics within default episodes for the variables of interest. Using our accounting framework we measure 62 default episodes in our dataset. The average length of the default episode is 10 years, but many defaults are shorter, about 35% of default episodes last less than or equal to 2 years.

Table 3: Dynamics during Default Episodes

Means (%)	Before	Beginning	Middle	After
Partial default	0	14	27	0
Official	0	8	22	0
Private	0	14	23	0
Debt to output (in %)				
Total	32	35	40	33
Official	17	18	24	19
Private	16	17	17	14
Debt service to output (in %)				
Total	5	5	4	5
Official	2	2	2	2
Private	3	3	2	3
Spreads	11	21	16	5
Output	3	1	-2	0

Notes: The dynamics of the default episode are averages across the 62 episodes in our sample for the variables of interest. "Before" is the period before the start of the episode; "Beginning" is the first period of the episode; "Middle" is the midpoint of the episode; "After" is the period when partial default returns to zero. Debt is reported relative to output; output is logged and linearly detrended and reported relative to the level before the episode. See notes in Table 1 for the definitions of the variables.

Table 3 reports the patterns of partial default, debt to output, and debt service for total debt, official debt, and private debt during default episodes. We also report the patterns of spreads and output. We report average statistics for these variables for the period before the start of the default episode, labeled *before*, the first period of the default episode, which we label *beginning*, the middle of the default episode, which we label *middle*, and the end of the default episode, which we label *end* and which is the first period when partial default returns to zero.[1] As in AMR, we also find that partial default tends to start smaller in the beginning and grows with the episode before returning to zero. Across types of debt, we also find this hump shape

1. We define the middle of the episode as the total length of the episode divided by 2, rounded to the nearest integer.

pattern, but the increase in partial default is faster than on official debt. As in AMR, we find that total debt to output features a hump-shaped pattern during the episode. Nevertheless, these dynamics are mainly driven by the dynamics of official debt. Official debt to output increases by about 7% during the episode and ends at levels higher than before the episode. Private debt to output increases by 1% during the episode and ends a bit lower than before the episode. Let us stress two main patterns: first private debt to output does not change substantially during default episodes and second official debt increases during the default episode and remains elevated after the episode.

The table also shows the patterns of debt service to output, spreads, and output. Surprisingly, debt service to output (total, official, and private) remains pretty elevated during the default episode and does not fluctuate much. It is only in the middle of the episode that debt service of official debt declines by 1%. Spreads feature a hump-shape pattern during episodes and output a U-shape pattern.

We conclude this section by summarizing our findings from our emerging market data. First, we document that official debt is large in emerging markets, and represents more than half of the external debt for governments. Second, we document that official and private debt grows with partial default, but the increase is sharper for official debt. Third, we find that official debt accounts for much of the increase in debt at the end of default episodes. In the next section, we develop a model with official and private sovereign debt and default to study theoretically the patterns of these debts during default episodes and rationalize these patterns.

3 Model

Consider an infinite horizon model of sovereign default with official and private debt. The sovereign has standard preferences over consumption

$$\mathbb{E}_t \sum \beta^t u(c_t)$$

where β is the discount factor and $u(c)$ is increasing and concave. The economy faces stochastic endowment z_t and can borrow from international official and private lenders. official debt is given by f_t and private debt is given by b_t. Debt contracts are perpetuities with coupon payments equal to $(r + \vartheta^f)f_t$ and $(r + \vartheta^b)b_t$ and that decay at rate $1 - \vartheta^f$ and $1 - \vartheta^b$, respectively. The sovereign can selectively partially default on each type of debt; the partial default decision for official debt is given by $d_t^f = [0, 1]$ and that for private debt is $d_t^b = [0, 1]$. A partial default of intensity d_t^a for debt of type $a = \{b, f\}$ means that the sovereign defaults on $d_t^a(r + \vartheta^a)a_t$ of the coupons due upon the partial default and also defaults $\mu^a d_{a,t}$ of all future debt coupons for that type of debt. Partial defaults lower resources for absorption, such that output depends on the endowment as well as partial defaults, $y_t(z_t, d_t^f, d_t^b) \leq z_t$ when $d_t^f > 0$ or $d_t^b > 0$. Partial default reduces the payments on the official or private coupons, but a fraction of the defaulted coupons accumulate and are due in the future. The fraction of defaulted coupons that accumulate is κ^f for official debt and κ^b for private debt.

The sovereign borrows new official loans ℓ_t^f at price q_t^f and private loans ℓ_t^b at price q_t^b to support consumption and pay off the existing debt due. The budget constraint for the sovereign is

$$c_t = y_t - (r + \vartheta^f)f_t(1 - d_t^b) - (r + \vartheta^b)b_t(1 - d_t^b) + q_t^f \ell_t^f + q_t^b \ell_t^b.$$

Consumption equals output net of the non-defaulted coupon payments of official and private debt and new loans. The structure of our perpetuity contracts and accumulation of coupons with partial default gives rise to the following laws of motion for each type of debt due

$$a_{t+1} = (1 - \vartheta^a)(1 - \mu^a d_t^a)a_t + \ell_t^a + \kappa^a d_t^a a_t \qquad \text{for } a = \{b, f\} \tag{2}$$

The laws of motion incorporate the coupons from the legacy debt that are not defaulted on $(1 - \vartheta^a)a_t(1 - \mu^a d_t^a)$, the new issuances ℓ_t^a and the accumulation of the defaulted coupons $\kappa^a d_t^a a_t$. Note that when $\kappa^a = (r + \vartheta^a) + \mu^a(1 - \vartheta^a)$ the entire stream of defaulted coupons accumulate with interest and are due in the future, which implies a full recovery equal to 1 upon default. Lower values for κ^a indicate that partial defaults result in a discharge of the debt.

The price for official and private loans are schedules that compensate lenders for the losses from default. As we will see below, this means that these schedules depend on the state the following period $\{f_{t+1}, b_{t+1}\}$ and on the endowment z_t because it is useful to forecast the endowment the following period. These schedules are $q^f(f_{t+1}, b_{t+1}, z_t)$ and $q^b(f_{t+1}, b_{t+1}, z_t)$.

Recursive Problem for Sovereign. The state variable for the sovereign includes the official and private debt and the endowment, $s = \{f, b, z\}$. The sovereign also takes as given the bond price functions for the two types of debt. Given these states and the bond price functions, the sovereign makes choices for partial defaults, official and private loans, and consumption to maximize its value

$$V(f,b,z) = \max_{d^f, d^b, \ell^f, \ell^b, c} u(c) + \beta \mathbb{E}\left[V(f',b',z')\right] \tag{3}$$

subject to the budget constraint

$$c = y(z, d^f, d^b) - (r + \vartheta^f) f (1 - d^f) - (r + \vartheta^b) b (1 - d^b) + q^f(f', b', z)\ell^f + q^b(f', b', z)\ell^b, \tag{4}$$

the accumulation equations of official and private debt in (2), and the restriction that partial default on official and private is bounded, $0 \leq d^f \leq 1$ and $0 \leq d^b \leq 1$. This problem results in decision rules for consumption, partial defaults, official and private borrowing, denoted by $\mathbf{c}(f,b,z)$, $\mathbf{d^f}(f,b,z)$, $\mathbf{d^b}(f,b,z)$, $\ell^{\mathbf{f}}(f,b,z)$, and $\ell^{\mathbf{b}}(f,b,z)$. We can use the decision rules for partial default and borrowing to determine the decision rule for next period's debts $\mathbf{f}'(f,b,y)$ and $\mathbf{b}'(f,b,y)$ as dictated by the laws of motion.

Loan Contracts. International lenders are competitive, discount the future at rate $1 + r$, and do not have any recourse in a partial default other than that dictated by the accumulation of defaulted coupons at rate κ^a. Bond prices are functions that depend on $\{f', b', z\}$ to compensate lenders for the expected loss of default which depends on these states. The bond price function for private and official loans, with $a = \{b, f\}$, satisfy

$$q^a(f', b', \tilde{y}) = \frac{1}{1+r} \mathbb{E}\left[(1 - \mathbf{d^a}') + ((1 - \vartheta^a)(1 - \mu^a \mathbf{d^a}') + \mathbf{d^a}' \kappa^a) q^a(\mathbf{f}'', \mathbf{b}'', \tilde{y}')\right]. \tag{5}$$

The expression for the bond price encodes the expected stream of payments per unit of the loan for the life of the perpetuity contract. The first term $(1 - \mathbf{d^a}')$ is the expected payment of the first coupon of the bond in the period following the issuance and it takes into account the potential partial default. The second encode that the perpetuity contract calls for the long-term promise to pay $(1 - \vartheta^a)$ fraction of the coupon the following period net of the reduction from the partial default $(1 - \mu^a \mathbf{d^a}')$. The third term $\mathbf{d^a}'\kappa^a$ takes into account that κ^a fraction of the defaulted coupons remain as future obligations. The future obligations arising from defaulted coupons and long-term promises contain default risk and a specific coupon structure, both of which are encoded in the continuation price $q^a(\mathbf{f}'', \mathbf{b}'', \tilde{y}')$. Importantly, this future bond price is evaluated at the equilibrium policy function given a particular choice $\{f', b'\}$.

Characterization of Partial Default. We now characterize the partial default decisions. Given the structure of our model, we can recast the sovereign problem in two stages. In the first stage, the partial default policies for official and private debt are determined given a state and any potential choices of future states $\{f', b'\}$. In the second stage, the sovereign makes its portfolio choices for official and private $\{f', b'\}$. For the first stage, in an interior optimum, the partial default policies are chosen to expand the budget set of the sovereign and satisfy the following conditions:

$$\begin{aligned} -y_{d^f}(z, d^f, d^b) =& f[(r + \vartheta^f) + q^f(f', b', y)((1 - \vartheta^f)\mu^f - \kappa^f)] \\ -y_{d^b}(z, d^f, d^b) =& b[(r + \vartheta^b) + q^b(f', b', y)((1 - \vartheta^b)\mu^b - \kappa^b)] \end{aligned} \tag{6}$$

The left-hand sides are the marginal costs of partial default for each type of debt in terms of output losses. The right-hand sides are the marginal benefits. Absent any accumulation of defaulted coupons, $\kappa^f = \kappa^b = 0$, nor default on legacy debt, $\mu^f = \mu^b = 0$, the marginal benefits are the expansion of resources from saving on the defaulted coupons for official debt $(r + \vartheta^f)f$ or private debt $(r + \vartheta^b)b$. With accumulation of coupons and default on legacy debt, however, the marginal benefit is the present value from the change in debt obligations that results from the partial default evaluated at market prices.

Note that low bond prices increase the incentives to default because they reduce the value of the defaulted coupons. At an interior solution, the partial default policy equates the marginal costs and benefits. However, partial default is bounded, $0 \leq d^a \leq 1$ for $a = \{f, b\}$. If the marginal costs strictly exceed the marginal benefits for any positive partial default for debt type a then $d^a = 0$; conversely if the marginal benefit exceeds the costs at $d^a = 1$, then default is full. Partial default incentives are bigger also when the values of official and private debt are large and when the accumulation factors of the partial default $\{\kappa^f, \kappa^b\}$ are low. Given this partial default policy, the sovereign chooses official and private debt to maximize its value taking as given the bond price functions.

4 Model Characterization

In this section, we simplify the model and characterize a few key properties of official and private debt contracts. To that end, we assume that official debt are perpetuities with $\vartheta^f = 0$ and private debt are short-term contracts $\vartheta^b = 1$. For simplicity, we also consider the case of linear utility, no accumulation of defaulted coupons, no default on legacy debt and a fixed default cost with any positive default. , such that $y_t = z_L$ if $d_t^f > 0$ or $d_t^b > 0$. We summarize these settings in the following assumption.

Assumption 1 (Simple Economy)**.** *In the simple economy,* $u(c) = c \geq 0$, $\vartheta^f = 0$, $\vartheta^b = 1$, $\kappa^f = \kappa^b = \mu^f = \mu^b = 0$, *and* $(1+r)\beta < 1$. *Absent default, productivity is constant* $z_t = z$, *and it falls to* z_L *if* $d_t^f > 0$ *or* $d_t^b > 0$. *Default does not prevent new borrowing.*

Default and Budget Sets. As is standard in sovereign default models, default incentives shape the price schedules for debt, and these in turn determine the supply of loans. Here we use our simplified model to characterize default incentives and the price schedules. A main objective is to characterize how official and private debt differ in terms of default incentives.

Under assumption 1, the recursive problem for the government is the following

$$V(f,b) = \max_{d^f \in [0,1], d^b \in [0,1], \ell^f, \ell^b, c} c + \beta\, V(f',b')$$

subject to $c \geq 0$ and the budget constraints. With no default, $d^f = d^b = 0$, the budget is

$$c = z - rf - (1+r)b + q^f(f',b')(f'-f) + q^b(f',b')b'.$$

Given that that default cost is fixed and independent on the intensity of the default, if the sovereign chooses to default, it fully defaults on the coupons of both debts, namely $d^f = d^b = 1$. The budget constraint with default is then

$$c = z_L + q^f(f',b')(f'-f) + q^b(f',b')b'.$$

Importantly, default does not preclude market access to borrowing or paying future debt. Default is a period-by-period decision that is costly only because it reduces resources if coupons are not paid. Given the setup, the sets of contracts available for official and private debt, namely $q^f(f',b')f'$ and $q^b(f',b')b'$, do not depend on whether the sovereign defaults or repays the coupons. This means that default will be chosen if it expands the budget set and that the default policies are:

$$d^f = d^b = \begin{cases} 0, & \text{if} \quad rf + (1+r)b \leq z - z_L \\ 1, & \text{otherwise.} \end{cases} \tag{7}$$

The default policies map into price functions. To analyze the impact of default policies on bond prices and budget sets, it is useful to consider using one of the types of bonds at a time. Suppose first that in period 1, the government uses only private debt. Given default policies, the bond price function for private debt depends on the one-period ahead budget, such that

$$q^b(f'=0,b') = \begin{cases} 1, & \text{if} \quad (1+r)b' \leq z - z_L \\ 0, & \text{otherwise} \end{cases}$$

The private loan that maximizes the budget, therefore is $b'_{\max} = \frac{z-z_L}{1+r}$ and the associated price is $q^b = 1$. If the sovereign chooses this maximal private debt contract, consumption expands by $\frac{z-z_L}{1+r}$. Given initial level of debt b_t, consumption in period 1 is

$$c_t = z - (1+r)b_t + \frac{z-z_L}{1+r} \qquad \text{for } t = 1.$$

With the maximal private contract, consumption in period 1 is expanded by $\frac{z-z_L}{1+r}$. Consumption in future periods is independent of the period 1 private contract.

Suppose now that the government only uses official debt. Unlike for private debt, the bond price function for official debt depends on all future default incentives and future borrowings. Consider a candidate official debt contract that gives the sovereign barely enough incentives to repay in the future. Given default decisions in (7), this contract has a coupon value that is equal to the cost of default, such that $rf_t = z - z_L$ for all $t \geq 2$.

An official contract at $t = 1$, that incorporates a transfer to the sovereign of $f = \frac{z-z_L}{r}$ and promises to pay $rf = z - z_L$ for $t \geq 2$, is the maximal contract that ensures repayment. In periods $t \geq 2$, the sovereign could change the coupon value rf by issuing loans ℓ_t^f. Nevertheless, any positive additional loans would carry a price of zero because they would trigger defaults on all of the official debt. This off-equilibrium strategy, therefore, could not expand the budget set of the sovereign for $t > 2$. The bond price function for official debt is therefore

$$q^b(f', b' = 0) = \begin{cases} 1, & \text{if} \quad rf \leq z - z_L \\ 0, & \text{otherwise} \end{cases}$$

With this contract at $t = 1$, given an arbitrary level of debt f_t, the path of consumption satisfies

$$\begin{aligned} c_t =& z - (1+r)f_t + \frac{z - z_L}{r} \qquad \text{for } t = 1 \\ c_t \leq& z_L \qquad\qquad\qquad\qquad \text{for } t \geq 2 \end{aligned}$$

With the maximal official contract, consumption in period 1 is expanded by $\frac{z - z_L}{r}$, and is reduced in future periods as the sovereign pledges future resources to servicing the official debt coupons.

The analysis comparing private and official loans gives our first result.

Lemma 1 (Official expands budget more)**.** *Under assumption 1,*

$$q(f'_{max}, b' = 0)f'_{max} = \frac{z - z_L}{r} > q(f' = 0, b'_{max})b'_{max} = \frac{z - z_L}{1 + r}.$$

This result arises because the official debt contract effectively constrains future governments from borrowing as the pledgeable resources, namely the default cost $z - z_L$, are already committed to the legacy official contract. The official contract can extract the present value of the pledgeable resources, $(z - z_L)/r$. Private debt, in contrast, does not constrain future governments from borrowing, and therefore it can extract only the one period ahead pledgeable resources $(z - z_L)/(1 + r)$. This difference implies that private contracts cannot replicate the paths of consumption that are possible with official contracts. This lack of replicability is in contrast with models without default, where the maturity structure of debt does not matter for allocations. When borrowers have default incentives, in general, maturity matters for allocations.[2] The common result in the sovereign default literature, however, is that short-term debt can replicate long-term debt contracts, but not vice-versa. In our partial default model, however, the result is opposite: official debt which is longer term, can replicate the allocations with only private debt, which is short-term. Next, we explore how our model's main differences relative to the standard model, namely borrowing during default and timing of default costs, shape these differences in replicability.

Assumption 2 (Permanent exclusion and output costs)**.** *In the simple economy, any positive default results in a permanent exclusion from financial markets and output costs.*

This is a standard assumption in the sovereign default literature in the tradition of Eaton and Gersovitz (1981), namely that default triggers a permanent cost in the form of exclusion from borrowing and output costs. Under assumption 2 the patterns of consumption change and therefore default decisions (7) are different. Consumption with default is equal to output net of the cost of default, $c = z_L$, while consumption during repayment depends on the policy functions for future official and private loans. Given the assumption of linear utility and impatience in (1), these policy functions are simple: the sovereign exhausts its borrowing capacity in every period. As above, let $\bar{b}$ and $\bar{f}$ be the maximum levels of private and official debt that prevent default. The following result shows that now both types feature the same debt limits.

Lemma 2. *Under assumption 2, official and private loans expand equally the budget*

$$q(f'_{max}, b' = 0) f'_{max} = \frac{z - z_L}{r} = q(f' = 0, b'_{max}) b'_{max} = \frac{z - z_L}{r}.$$

The standard assumption in the sovereign default literature of exclusion from financial markets after default is at odds with the empirical evidence documented above. During periods of default, sovereigns continue to participate in financial markets, and official loans in particular come in. We find interesting, however, that it is this assumption that leads to the possibility that short-term debt replicates long-term debt, in the presence of default. Under the more empirically relevant assumption in our baseline model, that default does not preclude borrowing, private shorter-term debt features more limited debt capacity than official longer-term debt.

2. See for example, Aguiar et al. (2019) and Arellano and Ramanarayanan (2012).

5 Quantitative Evaluation

This section presents the quantitative evaluation of our model. We first describe the parameterization of the model, which incorporates a moment-matching exercise that uses the panel data presented above. We then compare the implications of the model for additional moments that characterize default episodes and find that it delivers patterns comparable to the data. Finally, we evaluate the role of official lending for welfare and perform various counterfactuals which include voluntary swaps of the two different types of debt, as well as study the design of official debt contracts.

5.1 Specification and Parameterization

The utility function is $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$. The potential endowment follows a log normal AR(1) process $\log(\tilde{y}_t) = \rho \log(\tilde{y}_{t-1}) + \sigma_{\tilde{y}}\varepsilon_t$, where $|\rho| < 1$ and $\varepsilon_t \sim N(0,1)$. We discretize this process into 12 different states following Tauchen 1986.

The output costs of default are increasing and convex in the potential endowment, as in Arellano, Mateos-Planas, and Ríos-Rull 2023. These costs are realized only when $\tilde{y}$ is higher than a certain threshold $y^\star$, and then are linearly increasing in $\tilde{y}$ with slope ϕ. Similarly, output costs are increasing and convex in partial default d_b and d_f, with a slope parameter λ and a curvature parameter γ. The specific functional form for the output cost of default is given by:

$$y = \begin{cases} \tilde{y}(1-\lambda d_b^\gamma)(1-\lambda d_f^\gamma)(1-\phi(\tilde{y}-y^\star)) & \text{if } (d_b > 0 \text{ or } d_f > 0) \text{ and } \tilde{y} > y^\star \\ \tilde{y}(1-\lambda d_b^\gamma)(1-\lambda d_f^\gamma) & \text{elsewhere} \end{cases} \tag{8}$$

We calibrate the model at an annual frequency. We set some parameters to values from the literature and estimate others in a moment-matching exercise.

We set the annual international risk-free rate to 2%, consistent with yields from U.S. Treasury bills, and set the coefficient of risk aversion is 2, a standard value in the literature. We also set the default cost exponent γ to 2, a value close to that estimated in Arellano, Mateos-Planas, and Ríos-Rull 2023, for computational simplicity as it delivers a closed-form expression for partial default, equation (6). The autocorrelation and the standard deviation of the endowment process are set to match the persistence and volatility of output in the data.

In computing our model, we also incorporate discrete taste shocks following Dvorkin et al. 2018. These shocks slightly perturb the borrowing decision to achieve numerical stability and robust convergence in the computational algorithm. The parameter ϱ governs the relative importance of the taste shocks for the choice of b' and is set to $5e^{-5}$, which is the smallest value that guarantees convergence in both the official and private debt model.

The rest of the parameters are chosen to best fit moments in the data. The parameters that differ across the debt contracts are $\mu^a, \kappa^a, \vartheta^a$ for $a \in \{b, f\}$. It turns out that in our model given a decay parameter ϑ^a, the parameters that control default on legacy and the accumulation of defaulted coupons, μ^a and κ^a, matter only as the linear combination $\alpha^a = (1 - \vartheta^a)\mu^a - \kappa^a$ in the sovereign program. Therefore, in the moment matching exercise, we only estimate for each type of debt the decay ϑ^a and the net recovery factor α^a.

Table 4: Parameter Values

Risk-free interest rate	$r = 0.02$
Risk aversion coefficient	$\sigma = 2$
Endowment process	$\rho = 0.87$, $\sigma_{\tilde{y}} = 0.05$
Default cost exponent	$\gamma_1 = 2.0$
Taste shock	$\varrho = 5e^{-5}$
Moment-Matching Exercise	
Discount factor	$\beta = 0.96$
Debt contracts	
Decay parameters	$\vartheta^f = 0.08$, $\vartheta^b = 0.15$
Net recovery factors	$\alpha^f = 0.15$, $\alpha^b = 0.34$
Default Costs	
Cost based on partial default	$\gamma = 0.048$
Asymmetric endowment	$\phi = 1.2$, $y^\star = 0.99$

We perform a moment-matching exercise and estimate eight parameters. These parameters are the discount factor, the debt contract parameters, and the default cost parameters. We collect these parameters in $\Theta = \{\beta, \vartheta^f, \vartheta^b, \alpha^f, \alpha^b, \lambda, \phi, y^\star\}$. We target 9 moments on the distribution of debts and partial defaults. These moments are the means and standard deviations of debt-to-output ratios for total debt, official debt, and private debt; the means of partial default and the mean of the ratios of debt service to debt for official and private debt. Table 4 shows all the baseline calibration values for the parameters of the model.

All parameters affect all moments, but some moments are more informative of certain parameters. The ratios of debt service to debt are informative on the decay parameters, as when partial default is zero, these ratios are equal to $r + \vartheta^f$ and $r + \vartheta^d$ for official and private. The level of total debt informs the default cost parameters, as higher default is associated with higher debt capacity. The net recovery parameters matter for the relative levels of private and official as well as their standard deviations. The discount factor and the default cost parameters also matter for the average partial default and the volatility of the debts.

The resulting parameters controlling official and private contracts, namely $\{\vartheta^f, \vartheta^b, \alpha^f, \alpha^b\}$, imply two properties that are also consistent with external estimates. First, the exercise implies that the durations of official and private debt are 10 and 6 years, which are consistent with estimates in Arellano and Ramanarayanan 2012. Second, we find that official debt has lower recovery factors; the estimated parameters imply that the recovery for a one-period default is 45% for official debt and 64% for private debt.[3] The more concessional nature of official debt is consistent with the findings of Schlegl, Trebesch, and Wright 2019 and Horn, Reinhart, and Trebesch 2020.

5.2 Baseline Quantitative Results

We start with the results from our moment matching exercise. Table 5 reports the model's implications for our target moments as well as for additional moments. The model statistics come from a long simulation of 200000 periods, after discarding the first 10000 observations. The model matches well mean total debt and the breakdown of official and private. In the model and data, debt to output is on average close to 30% and about two-thirds of that debt is official. The standard deviations of the debts are also in line with the data, although in the model debts are a bit less volatile than in the data. In terms of partial default, which in our model and data is measured as total debt in arrears due relative to total debt due, $\frac{d_t^b(r+\vartheta^b)b_t + d_t^f(r+\vartheta^f)f_t}{(r+\vartheta^b)b_t + (r+\vartheta^f)f_t}$, is on average 36% conditional on positive in the model conditional on positive, close to the data average of 32%. Finally, the ratios of debt service to debt of 8 and 14 in the model are similar to those found in the data.

3. As we explain below, the recoveries for an average default episode will be lower because they last longer than one period.

Table 5: Model Fit (9 moments, 8 parameters)

	Data	Model
Target moments		
Total Debt		
mean	32	33
st. dev.	18	14
Official Debt		
mean	20	21
st. dev.	12	9
Private Debt		
mean	13	13
st. dev.	8	6
Partial Default		
Total	32	36
Debt service to debt		
Official	11	8
Private	19	14
Other moments		
Debt service to output		
Total	3.6	3.3
Official	1.6	1.6
Private	1.9	1.7
Partial Default		
Official	31	40
Private	35	32
Corr (d^f, d^b)	62	98
Corr (f, b)	42	86

The table also reports some additional unconditional first and second moments. The model resembles the data in the ratios of debt service to output and average official and private partial default, although in the model official partial default is somewhat higher than the data. In the data, the correlations between official and private partial default and between official and private debt are positive, as implied by the model, although in the model these correlations are stronger.

Table 6: Moments Conditional on Partial Default

	Data		Model	
	No default	Partial default	No default	Partial default
Debt to output	23	44	25	42
Official	13	29	16	25
Private	11	15	9	17
Private spreads	4	8	2	5
Partial default	0	32	0	36

5.2.1 Partial Default Bins

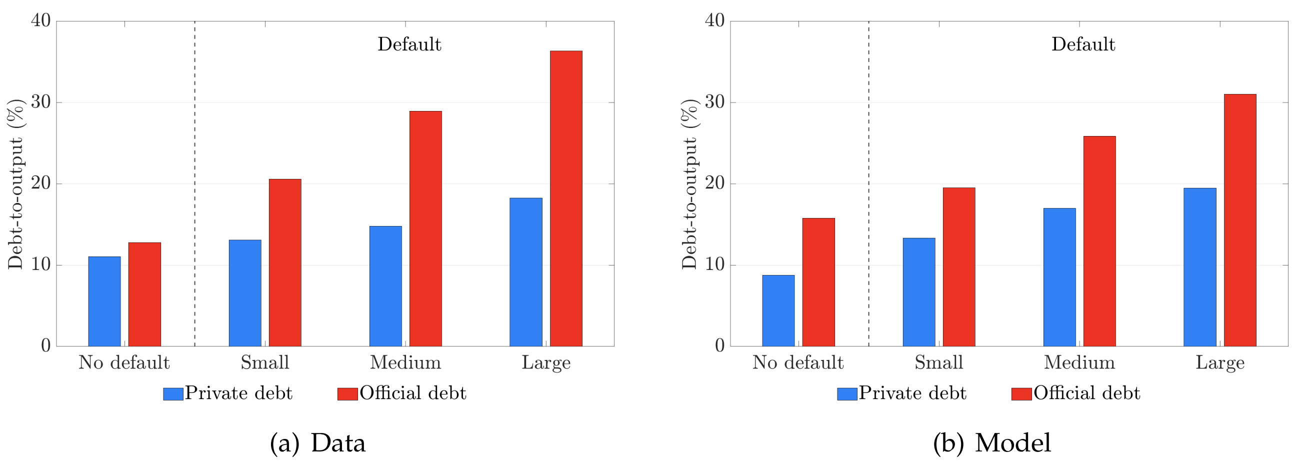
The successful moment-matching exercise illustrates that the model resembles the data in the key first and second moments. We now analyze how the main variables of interest change with positive partial default. We explore the patterns for total debt-to-output as well as official, and private debt, and private spreads. In Table 6 we compare the means of these variables across bins with and without partial default across model and data. The *No default* bin corresponds to the observations with zero partial default, whereas the bin labeled *Partial default* corresponds to periods with positive values for partial default.

The model generates higher debt-to-output ratios with partial default as in the data. During periods of positive default, total debt-to-output is 17% higher than in periods of no default (42% vs. 25%). This increase is similar to that observed in the data, which equals 19%. Official debt increases more with default than private debt in both model and data. In the model, however, these differences are a bit less accentuated than in the data. Spreads also increase with default. Spreads on private debt are about 3% higher when partial default is positive, comparable to the increase of 4% in the data.

As in the empirical analysis in Section 2, we further examine the patterns of official and private debts across finer partial default bins. In Figure 5, we report mean official and private debt across 4 bins, the same ones we used in Section 2. As before, the bars represent the averages of official and private debt-to-mean output across bins; the red bars correspond to official debt and the blue bars correspond to private debt; the left panel is the data and the right panel is the model. The figure illustrates that in the model, as in the data, debt increases monotonically with partial default. When partial defaults are small, under the 25 percentile of the distribution, debt to output is higher than when default is zero. Large defaults, over the 75 percentile of the distribution, feature the highest debt to output. The increase is more accentuated for official debt than for private debt in both the model and data.

This analysis illustrates that the model captures well the conditional patterns of debts across defaults across the limiting distribution. Periods of more intense defaults are associated with higher debt to output, especially so for official debt. Next, we analyze dynamics within default episodes. We illustrate that the higher debt during default can be understood as economies accumulating the defaulted debt as arrears and borrowing more loans.

Figure 5: Official and Private Debt across Default Bins



Notes: The figure plots mean debt (relative to mean output) across default quartiles in the limiting distribution for the models with official and private debt.

5.2.2 Default Episodes

We now study default episodes. As in the data, we use the simulated time series data to track default episodes. As described above, a default episode is a period of time with continuous positive partial default. We will study how debts, partial defaults, and spreads evolve across default episodes. We will also compute the length of the episode and the haircuts that each of the type of lenders get on average from the default episode. Table 7 reports the results of the model and compares them to data.

Default in the model is persistent, which gives rise to long default episodes. The average length of default episodes in the model is 7 years, which is of similar orders of magnitude to the data average of 10 years. We use the model's default episodes to compute the haircuts that lenders get. As is standard in the literature, haircuts are derived from comparing present values of streams of defaulted debt versus streams of new restructured debt. Haircuts tend to increase with longer default episodes because the sovereign's net recovery factors α^a compound with repeated defaults.[4] In the model, the average haircut from default episodes for official lenders is 58% and for private lenders it is 40%. These haircuts are very comparable to those in data, as reported by Schlegl, Trebesch, and Wright 2019.[5]

As reported in Table 7, during default episodes total debt features a hump-shaped pattern in the model, as in the data. In the data total debt to output increases about 8% from the year before the episode up to the middle of the episode (i.e. 39% minus 31%), which is equal to the 8% increase observed in the data. In the model, about half of the increase is official, while in the data the increase is official accounts for a larger fraction. We have found that in the model, if debts have equal recovery factors, then the official long-term debt tends to increase more during default episodes relative to private shorter-term debt. We are in the process of understanding further how these two differences across debt matter for rationalizing the patterns in default episodes.

Table 7: Default Episodes

	Dynamics of Debt				Haircuts
	Before	Beginning	Middle	After	
Data					
Total	32	35	40	33	
Official	17	18	24	19	60
Private	16	17	17	14	40
Model					
Total	31	35	39	36	
Official	21	23	25	23	58
Private	10	12	14	13	40

Notes: The mean length across default episodes is 10 years in the data and 7 years in the model. See the notes in Table 3 for further details.

4. See Appendix A for the formulas of haircuts.

5. We should note that in that work, estimates for private haircuts are computed from bond restructurings while haircuts from official lenders are computed from restructurings from Paris Club lenders. In our data, private debt also incorporates bank loans and official debt incorporates multilateral organizations.

6 Counterfactuals

We now use our baseline quantitative model to perform three counterfactuals. In the first counterfactual, we study the role of official debt, by comparing our baseline with an economy without official debt. We find that the existence of official debt is welfare improving for indebted economies. In the second set of counterfactuals, we study the design of official debt contracts by comparing our baseline with economies that feature official debt that is shorter-term and less concessional. This environment is motivated by different loan programs offered by multilateral organizations, such as the Stand-By Agreement (SBA) or the Short-Term Liquidity Line (SLL) of the International Monetary Fund (IMF), as well as the findings in Schlegl, Trebesch, and Wright 2019 that the IMF has very few of its loans in arrears. In the third counterfactual, we evaluate the feasibility of voluntary swaps across official and private debt, for lenders and the sovereign. We find that swaps of private debt for official debt are feasible for economies that are highly indebted with private loans.

6.1 Role of Official Debt

In this section, we explore the positive and welfare implications of official debt. To this end, we study an economy with two debts that have the same duration and recovery factor as the private debt of the baseline. This exercise exemplifies an economy with only private debt.

The second column of Table 8 compares the main summary statistics of the economy with only private debt to the baseline economy with official debt. Welfare gains and losses are expressed in consumption equivalence units relative to the baseline economy.

With only private debt, the debt levels are smaller, and partial default is about 5% lower. Private spreads are higher, and the consumption volatility increases relative to the baseline economy. The table also presents welfare results for particular states. When output is at the mean and the economy has no debt, the existence of official lending is slightly welfare-reducing, but when debt is at the mean of the limiting distribution of the baseline, official debt improves welfare. Eliminating official debt in this state reduces welfare by 0.07%. The welfare losses double when the economy has high debt, with levels of official and private equal to 37 and 23% of output, respectively.

Table 8: Counterfactual

	Baseline	Official Debt		
		Equal to Private	Short + Less concess.	Long + Less concess.
Official debt	22	13	7	39
Private debt	12	13	14	12
Partial default	38	33	26	47
Spreads	3.7	3.9	4.2	3.5
Consumption st. dev	0.95	0.97	0.98	0.94
Haircut official debt	58	41	30	31
Haircut private debt	40	41	41	40
Welfare (CE, %)				
No debt (mean $\tilde{y}$)	0.00	0.004	0.029	0.071
Mean debts b, f (mean $\tilde{y}$)	0.00	-0.07	-0.124	0.046
High debts b, f (mean $\tilde{y}$)	0.00	-0.14	-0.330	0.007
No debt (low $\tilde{y}$)	0.00	0.010	0.071	0.138

6.2 Design of Official Debt

We now study how the properties of official debt affect the economy and the welfare implications. To this end, we change the duration of official debt and the recovery factor. We find that contracts with high debt duration and high recovery factors tend to give the highest welfare to the sovereign.

Although official debt tends to be of longer maturity and is more concessional, certain official loans are quite short-term and with very little concessional characteristics. The SBA and SLL of the International Monetary Fund and the Federal Reserve swap lines are official loans that are very short-term and designed with very little default risk in mind. Motivated by these programs, we evaluate the implications of making all official loans shorter-term and with a high recovery factor. In Table 8, column 3, we present results for our model for the case when official loans have a maturity of two years and a higher recovery factor equal to that for private loans. In this case, the economy can sustain about one-third of the official debt in the baseline (7% vs. 22%), and partial default is 12% lower. However, consumption volatility is higher and spreads increase by half of a percentage point. In terms of welfare, we find mixed results: in low-debt states, shorter-term debt with high recovery rates tends to be more beneficial to the sovereign relative to the baseline official loans. An important exception is high-debt states. During high debt states, the sovereign prefers the baseline contracts because higher haircuts allow for a larger reduction of the debt burden, which ex-post is beneficial.

Next, we explore the implication of moving only recovery factors, while keeping the duration as in the baseline. Table 8, column 4, presents these results. In this case, this economy can sustain higher levels of debt in equilibrium. Spreads decrease by 0.2% compared to the baseline, and consumption volatility is also lower. Welfare gains range from 0.007% in states of high debts and mean output to 0.13% when the economy has no debt and low output. The main takeaway from this exercise is that official debt of long duration and high recovery factors tends to be the best for the sovereign.

6.3 Voluntary Swaps

Finally, we evaluate the possibility of a voluntary swap program, where the country and its official and private lenders agree to a debt exchange from an initial level (b, f) to new levels

$(\hat{b}, \hat{f})$. As discussed above, official debt gives greater debt capacity and increases welfare for highly indebted economies. Since in our baseline model each lender contracts with the country independently, there is room for swaps of private bonds for official bonds upon the realization of a state. A voluntary swap exchange is feasible if the following two conditions hold:

$$V(\hat{b}, \hat{f}, y) \geq V(b, f, y) \tag{9}$$

$$q^b(\hat{b}, \hat{f}, y)\hat{b} + q^f(\hat{b}, \hat{f}, y)\hat{f} \geq q^b(b, f, y)b + q^f(b, f, y)f \tag{10}$$

Condition (9) indicates that the value of the country must weakly increase with the swap program. Similarly, Condition (10) implies that the total value of the debt must be at least as high as the value in the absence of the swap.[6]

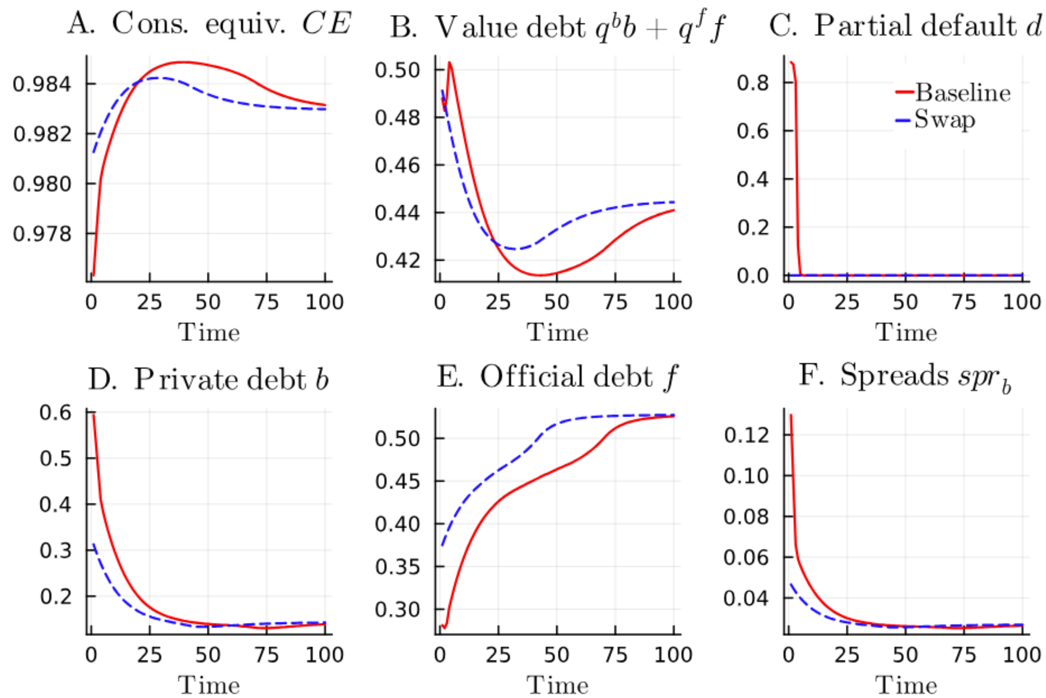
We use our quantitative model to explore whether there is room for a voluntary swap and to shed light on the macroeconomic implications of the proposed debt exchange. We find that across the state space of the baseline model, there are many states (y, f, b) where conditions ((9) and (10) hold for at least one new level $(\hat{f}, \hat{b}, y)$. In these states, a voluntary swap exchange is feasible. Moreover in the limiting distribution of the economy, voluntary swaps are feasible in about 1% of the distribution.

To gain more insights into these dynamics, Figure 6 plots the dynamics starting in one of these states where swaps are feasible. The blue dash lines in the figure are the paths for the equilibrium economy, when we keep output constant at the initial level. The red solid lines are the time paths after implementing a voluntary swap. We choose the new debt levels of the swap $\hat{f}m\hat{b}$ to be those that maximize the welfare of the sovereign, within the set of feasible levels. The figure compares the time paths for the country's welfare, total value of the outstanding debt, partial default, debts, and spreads in our baseline economy with those of an economy that implements a voluntary swap in the initial period.

Panel A shows the time path for the consumption equivalence welfare. In the initial period, the country is strictly better off with the swap, as welfare increases by 0.5% compared to the baseline. Along the transition, the swap economy exhibits a less volatile welfare, induced by a smoother consumption path. Panel B shows the total value of the debt. Since lenders

6. In setting these conditions, we are implicitly assuming that the official lender can act as a big player that values the initial debt at initial market prices.

Figure 6: Dynamics with a Voluntary Swap



are willing to participate in the swap, the initial point is higher under the swap economy. In the baseline, the value increases during the first periods, overshoots, and then settles at a level below the one of the swap. Partial default paths are compared in panel C. During the first periods of the transition, the baseline economy experiences positive partial default, inducing some initial default costs. In contrast, the voluntary swap eliminates the debt crisis, as the sovereign does not default in any period of the simulation. Panels D and E display the paths for official and private debt. The implementation of the swap reduces the initial level of the private debt while increasing the one for official. Private debt decreases throughout the transition and settles at a lower level, allowing both economies to run up their official debt levels. Private spreads decrease by 0.08% with the swap, as shown in panel F.

The feasibility of voluntary swap of private for official debt confirms that the properties of official lending are very beneficial for highly indebted sovereigns. The new insights here are injections of official loans can help alleviate the costs of debt crises by eliminating costly defaults. Moreover, these injections can also benefit private lenders and deliver Pareto improvements to the decentralized economy.

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A Haircuts Formulas

Here we develop formulas for computing haircuts for each type of debt a across episodes. Consider a default episode of length $J - j + 1$ periods with $d_t^a > 0$ for $t = j, .., J$ with $d_{j-1}^a = 0$ and $d_{J+1}^a = 0$. The haircut for debt a for this episode is

$$\text{Haircut}_j^a = 1 - \frac{\text{Restructured Debt}_j^a}{\text{Defaulted Debt}_j^a} \tag{11}$$

where the defaulted debt is the present value of all defaulted coupons and satisfies:

$$\text{Defaulted Debt}_j = \sum_{k=1}^{J-j+1} \frac{d_{j+k}^a a_{j+k}}{R^{k-1}} (r + \vartheta + \mu(1 - \vartheta^a))$$

and the restructured debt is the present value of all restructured coupons and satisfies

$$\text{Restructured Debt}_j = \sum_{k=1}^{\infty} (r + \vartheta^a) \frac{(1 - d_{j+k}^a) n_{j+k}^a}{R^k}$$

where the restructured coupons n_k contain the streams of accumulated debt and equal

$$\begin{aligned}
n_{j+1}^a =& k^a d_j^a a_j \\
n_{j+2}^a =& (1 - \vartheta^a) n_{j+1}^a + k^a d_{j+1}^a a_{j+1} \\
&... \\
n_{J+1}^a =& (1 - \vartheta^a) n_J^a + k^a d_J^a a_J \\
n_{J+1}^a =& (1 - \vartheta^a) n_{J+1}^a \\
n_{J+2}^a =& (1 - \vartheta^a) n_{J+2}^a \\
&...
\end{aligned}$$