Empirical Investigation of a Sufficient Statistic for Monetary Shocks*

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Abstract

In a broad class of sticky-price models the non-neutrality of nominal shocks is captured by a simple sufficient statistic: the ratio of the kurtosis of the size-distribution of price changes over the frequency of price changes. We extend previous results beyond once and for all shocks, to improve the match between the model and empirically estimated monetary shocks with a transitory predictable component. We test the sufficient statistic proposition using data for a large sample of products representative of the French economy. We use the micro data to measure kurtosis and frequency for about 120 PPI industries and 220 CPI categories. We use a Factor Augmented VAR to measure the industries' response to monetary shocks, under alternative identification schemes. The estimated degree of non-neutrality correlates with the kurtosis and the frequency consistently with the prediction of the theory. Several robustness checks are discussed.

Keywords: Impulse response functions, Monetary Shocks, Generalized Hazard Function, Sticky prices, Sufficient statistic

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1 Introduction

A recent result identified a sufficient statistic for monetary shocks: the cumulative response of output to a once-and-for-all monetary shock is proportional to the ratio of the *kurtosis* of the steady-state distribution of price changes over the *frequency* of price changes. The result was established in Alvarez, Le Bihan, and Lippi (2016) for the sticky price model of Nakamura and Steinsson (2010), that nests as special cases two workhorse of macroeconomics: Calvo (1983) and Golosov and Lucas (2007). The result was extended by Alvarez, Lippi, and Oskolkov (2022) to a broader class of state-dependent models using the generalized hazard function setup of Caballero and Engel (1993, 1999). Alvarez, Lippi, and Paciello (2016) showed the same sufficient statistic to hold in models where firms follow time-dependent rules as in Reis (2006). Recent results by Baley and Blanco (2021) seek sufficient statistics for setups with a non-negligible drift. This extension is useful for applications to investment problems or to economies with high inflation. Given the multitude of theoretical setups that produce this prediction, Leahy (2016) considered the empirical test of the sufficient-statistic proposition a priority for this research program.[1] This paper takes up that challenge and presents a test of the sufficient-statistic proposition for monetary shocks in a low inflation environment.

We begin by extending the theoretical framework, developed for once-and-for-all permanent shock, to accommodate shocks with a predictable transitory component. Such an extension is important to map the model to the data, where nominal interest rate shocks are typically mean reverting. It is also, per se, an original contribution of the present paper. We handle this problem using the mean-field-game setup developed by Alvarez, Lippi, and Souganidis (2022). The analytical solution method revolves around a linearization, along the lines explored by Boppart, Krusell, and Mitman (2018) in numerical work to study MIT shocks. The results show that the sufficient statistic proposition remains informative about monetary non-neutrality even in the presence of mean-reverting shocks. We then test the sufficient-statistic predictions using micro data for a large number of firms, representative of the French economy, underlying the producer price indices (PPI) and the consumer price indices (CPI). The test is made of three steps. We first estimate

[1] He wrote: "I would not expect this equation to fit the data perfectly, but it would be very nice to know if these statistics are at all informative" (page 462 of Leahy (2016)).

the sectoral responses to a monetary shock for about 120 PPI industries and 220 CPI categories, using a Factor Augmented VAR in the vein of Bernanke, Boivin, and Eliasz (2005) and Boivin, Giannoni, and Mihov (2009). We summarize the extent of the non-neutrality using the cumulative impulse response of the sectoral prices (CIR^P). As the sufficient statistic proposition concerns the cumulated response of *output*, we use the theory to derive the testable implications for the cumulated response of *prices*. This allows us to increase the number of cross-sectoral observations, since output data are scarce relative to pricing data, and to map the theoretical prediction into a metric that is more robust.[2] The second step consists in using the micro data underlying the sectoral data to measure the cross sectional moments of the distribution of price changes in the different sectors. In the third step, we inspect the relationship between the CIR^P and the cross-sectoral moments under the restrictions implied by the theory.

The results consistently show that the data do not reject the predictions of the theory across a variety of tests, specifications, and robustness exercises. Both the frequency and the kurtosis appear as statistically significant factors in accounting for the cross-sectional heterogeneity of the estimated CIR^P for both the PPI data as well for the CPI data. The sign and magnitudes of the estimated coefficients are consistent with the predictions of the theory in the specification where the variables enter the regression as a ratio, as the theory prescribes, as well as in an unrestricted specification where both variables are entered as separate regressors. Moreover, "placebo" tests show that moments not suggested by the theory, such as the size, standard deviation and skewness of price changes, are not correlated with the CIR^P. In addition, the results are robust to allowing in various ways for measurement errors, an important concern when micro price data are used. When we compare results for PPI and CPI products, we find that the results for PPI are more robust than the results for CPI products. In the robustness analysis, we find that when removing products with frequent sales and substitutions (in particular, food, clothing and furniture), the CPI results align more closely with the ones obtained for PPI products. This is consistent with the fact that the model underlying the sufficient statistic result assumes no seasonal sales. It should not be surprising that, given the simplicity of the model and the many measurement issues involved, the variables suggested by the theory explain only a fraction of the cross sectional differences in

[2]The output response depends on sector specific elasticities that require additional information for the test.

the non-neutrality. But it is noticeable that, across of broad range of specifications, both kurtosis and frequency are related to non-neutrality in a way that is aligned with theory and that both variables are statistically significant.

At a high level, this paper relates to the voluminous applied literature that analyzes the implications of price-setting patterns, in particular cross sectoral heterogeneity, for the propagation of shocks.[3] The specific novelty of this paper is to test the sufficient-statistic proposition for monetary shocks, using the restrictions implied by the theoretical model. The theory guides our empirical analysis: it identifies the variables of interest, how they enter the test, and shows how to interpret sign and magnitude of the estimated coefficients. Previous studies highlighted the importance of the frequency of price changes as a factor behind the cross-sectoral response to an aggregate shock, e.g. Nakamura and Steinsson (2010); Gorodnichenko and Weber (2016); La'O and Tahbaz-Salehi (2020). The sufficient statistic proposition that we analyze supplements the predictions for the role of frequency with a prediction for the role of kurtosis, which indeed our data confirm to be relevant. A related analysis is developed by Hong et al. (2020), who inspect the correlation between the response of sectoral producer price indices in the United States and several cross-sectional moments of the distribution of price changes. The authors' aim is to develop a broad empirical investigation on the determinants of sectoral responses to monetary policy shocks, without focusing on a specific theory behind the empirical analysis. The lack of a tight link between the theory and the empirics prevents such results from providing a rigorous test of the sufficient statistic proposition.

The paper is organized as follows. Section 2 recalls the sufficient-statistic result and extends it to mean-reverting monetary shocks with a predictable component. Section 3 derives the theoretical restrictions to be tested on the data. Section 4 uses micro and sectoral data to measure the key ingredients needed to test the theory: (i) the sectoral response of prices and output to monetary shocks (ii) candidate sufficient statistics, i.e. several cross-sectoral micro moments. Section 5 presents the baseline results of the test using cross sectional data. Section 6 investigates the robustness of our findings using a number of alternative measures and specifications. Section 7

[3]See, e.g., Bils and Klenow (2004); Burstein, Eichenbaum, and Rebelo (2005); Carvalho (2006); Bouakez, Cardia, and Ruge-Murcia (2009); Imbs, Jondeau, and Pelgrin (2011); Cavallo and Rigobon (2016); Cavallo (2018, 2019); Amiti, Itskhoki, and Konings (2019); Bonomo, Carvalho, Kryvtsov, Ribon, and Rigato (2020); Carvalho, Lee, and Park (2021); Dedola, Kristøffersen, and Züllig (2021); Auer, Burstein, and Lein (2021).

concludes and discusses avenues for future research.

2 A sufficient statistic for monetary shocks

This section reviews the sufficient-statistic result for monetary shocks in an economy with sticky prices. Section 2.1 summarizes the sufficient-statistic result on the propagation of a permanent monetary shock. Section 2.2 provides more details on the foundations of the sticky-price model and extends the setup to accommodate shocks with a predictable transitory component. The extension is important for the empirical application using actual monetary shocks, such as mean reverting interest rate shocks. We solve for the economy's response to such dynamical shocks, and show that while the sufficient-statistic result does not hold exactly in such cases, the result remains close to the benchmark case of the permanent shock, so that the sufficient-statistic remains informative.

2.1 The Sufficient Statistic Result

Consider a firm's i (log) markup in industry $j = 1, \ldots, n$, defined as the price over the unit labor cost: $\mu_{ij}(t) \equiv \log \frac{P_{ij}(t)}{W(t)Z_{ji}(t)}$, where $P_{ij}(t)$ is the price of firm i in industry j, $W(t)$ denotes the aggregate nominal wage and $1/Z_{ij}(t)$ is the firm's i labor productivity. Assume the firm faces a demand with constant elasticity, and let μ_j be the time-invariant optimal markup. Define the "markup-gap" for firm i in industry j as

$$g_{ij}(t) \equiv \mu_{ij}(t) - \mu_j = x_{ij}(t) - \mathcal{W}(t) \quad \text{where} \quad x_{ij}(t) \equiv \log \frac{P_{ij}(t)}{\bar{W} Z_{ij}(t)} - \mu_j \text{ and } \mathcal{W}(t) \equiv \log \frac{W(t)}{\bar{W}} \quad (1)$$

Assume that $\log Z_{ij}$ follows a driftless diffusion, so that each firm is hit by idiosyncratic shocks $dx_{ij} = \sigma d\mathcal{B}_{ij}$ where each $\mathcal{B}_{ij}$ is a standardized Brownian motion, independent across i and j. The presence of sticky prices, and the stochastic productivity shocks, imply that the firm markup will not be equal to μ_j at every moment. We assume the initial conditions are such $\int \log Z_{ij}(t)di = 0$ for all industry j. The steady state nominal wage is given by $\bar{W}$, and $\mathcal{W}(t)$ denotes the deviations from the steady state that follow an aggregate shock.

In the sticky price (CalvoPlus) economy the aggregate output of industry j, in deviation from steady state, is proportional to the cross-section mean of the price gaps. Resorting to equation (1) we have

$$Y_j(t) \equiv -\frac{1}{\epsilon_j}\int g_{ij}(t)di = \frac{1}{\epsilon_j}\left(\mathcal{W}(t) - \int x_{ij}(t)di\right) \tag{2}$$

where ϵ_j is the industry-specific income elasticity – further discussed in Section 2.2– which depends on the "demand side" of the economy, i.e. it is independent of the firm price setting decisions. Define the industry j aggregate price $p_j(t) \equiv \int \log P_{ij}(t)di$, with steady state value $\bar{p}_j = \mu_j + \log \bar{W}$, we have:[4]

$$p_j(t) = -\epsilon_j Y_j(t) + \log W(t) = \int x_{ij}(t)di + \log \bar{W} \tag{3}$$

Note that $p_j(t)$ is determined by $W(t)$ and $\{x_{ij}(t)\}$, and that it does not depend on ϵ_j.

A permanent monetary shock. We consider an economy in steady state with an invariant distribution of price gaps x, and analyze the effect of an unexpected once-and-for-all monetary shock of size $\delta > 0$. The shock immediately (and permanently) raises the nominal wage $\bar{W}$, so that all firms' markups fall by δ (log points). Note that in this case $\mathcal{W}(t) \equiv 0$. The shock triggers a dynamic response of output, following equation (2). Let the cumulative impulse response (CIR^{Y_j}) of output be:

$$CIR^{Y_j}(\delta) = \int_0^\infty Y_j(t;\delta)\,dt \tag{4}$$

where $Y_j(t;\delta)$ is the aggregate output t periods after the shock δ, measured in deviation from the steady state output. The variable CIR^{Y_j} is a convenient statistic that summarizes with a single number the overall impact of the monetary shock on industry j.

The sufficient-statistic result establishes that the cumulated output response following a small nominal shock δ is

$$CIR^{Y_j}(\delta) = \frac{\delta}{\epsilon_j}\frac{Kurt_j}{6\,Freq_j} + o(\delta^2). \tag{5}$$

The result states that the cumulated output response to a monetary shock is accurately ap-

[4] Note p_j is the first order approximation of the ideal index: $\log P_j(t) = \frac{1}{1-\eta_j}\int P_{ij}(t)^{1-\eta_j}di \approx \int \log P_{ij}(t)di$.

proximated by the ratio of the kurtosis of the size distribution of price changes ($Kurt_j$) to the frequency of price changes $Freq_j$. The approximation is accurate up to second order terms. The result in equation (5) is striking. It holds in a large class of inherently different models, from time dependent models a la Calvo, to canonical menu-cost models a la Golosov-Lucas, intermediate cases such as the Calvo-Plus by Nakamura and Steinsson (2010) or inherently random-menu cost models such as those of Caballero and Engel (1993, 1999).

The effect of the frequency is well understood: a higher mean frequency of adjustment implies that adjustment is faster and hence the economy is more flexible (a smaller output effect). The effect of kurtosis is more subtle: it indicates that two industries with the same frequency can have substantially different flexibility. Kurtosis captures the fact that in an economy with heterogeneous agents the response to an aggregate shock depends on the shape of the cross sectional distribution of these agents, a fact emphasized in several papers by Caballero and Engel. Consider for an instance an economy where price setting is staggered every T periods, a la Taylor, and one where price setting is follows a Calvo rule with an average duration equal to T. These economies have the same frequency of price changes but the Calvo economy features a tail of "late adjusters", firms that even long after T periods have not adjusted their price following the shock. Such an effect is captured by the kurtosis of the size of price changes, even in models where a time-dependent rule is followed as in Carvalho and Schwartzman (2015); Alvarez, Lippi, and Paciello (2016). Intuitively, kurtosis summarizes the degree of cross sectional heterogeneity in the timing and size of price setting behavior. Equation (5) proves that this feature is important for the propagation of monetary shocks.

Key Assumptions and Limitations of the Sufficient Statistic Result. Three assumptions are key for equation (5) to hold. The first one is that the model has no inflation, so that several model objects display symmetry properties. While the assumption of zero inflation might seem restrictive, we argue that it provides a good approximation to models where inflation is low. The second key assumption is that upon adjustment the firm completely closes the price gap, i.e. that x is reset to zero. This assumption is violated in models with high inflation, or in models with "price plans" or "sales", such as in Eichenbaum, Jaimovich, and Rebelo (2011). In such cases equation (5) is not a good summary of the impulse response and other methods can be used to approximate CIR^Y. A third assumption is that x follows a Brownian motion. This allows us to use stochastic calculus to analytically characterize the firm's optimal policy and the associated cross sectional distribution of desired adjustments. In a model with leptokurtic shocks, such as Midrigan (2011), one cannot prove that kurtosis and frequency are enough to summarize the CIR^Y. However, for moderate deviations from the Brownian benchmark, consistent with the data on the distribution of firms' nominal shocks, the formula continues to provide a useful benchmark (see Section 5 in Alvarez, Le Bihan, and Lippi (2016) and the numerical results in Gautier and Le Bihan (2022)).

We compute impulse responses by perturbing the stationary state of the model. Hence, what we compute is the "expected impulse response", i.e. averaging the initial conditions, each of which is an entire distribution. Our setup could be used to study impulse responses where the initial condition is not the stationary distribution. This has been done in other models such as the empirical analysis by Caballero, Engel, and Haltiwanger (1997) in the context of employment, and the theoretical characterization for a price-setting model by Caplin and Leahy (1997). In our context such analyses would require more extensive data, to fit the CIR at different aggregate states of the economy, as well as new theoretical results to characterize the state-dependent CIR.

2.2 A model with predictable transitory shocks

This section describes the foundations of the model and introduces monetary shocks with a predictable transitory component. These shocks amount to a perturbation of the entire dynamic path of the aggregate nominal costs. The goal is to explore the robustness of the sufficient statistic result beyond the case of the once-and-for-all shock. While our interpretation will be in terms of a monetary shock with a persistent component, an equivalent interpretation can be given in terms of sticky wages, gradually changing through time.

2.2.1 The household side, wages, money and interest rates.

The representative household preferences in the Golosov and Lucas (2007) model, augmented to have n industries, are:

$$\int_0^\infty e^{-\rho t}\left[\sum_{j=1}^{n}\frac{c_j(t)^{1-\epsilon_j}}{1-\epsilon_j}-\alpha L(t)+\log\left(\frac{M(t)}{P(t)}\right)\right]dt \text{ and } c_j(t)=\left[\int_0^j A_{ij}^{\frac{1}{\eta_j}}c_{ij}(t)^{1-\frac{1}{\eta_j}}\right]^{1-\frac{1}{\eta_j}} \tag{6}$$

where $c_j(t)$ is a CES aggregate across the varieties sold by the firms in industry j, $L(t)$ is labor at time t and $M(t)$ nominal money holdings. The household problem has the following first order conditions: $e^{-\rho t}\alpha = \lambda Q(t)W(t)$, $e^{-\rho t}\frac{1}{M(t)} = \lambda Q(t)R(t)$, and $e^{-\rho t}c_j(t)^{-\epsilon_j} = \lambda Q(t)P_j(t)$, where $W(t)$ is the nominal wage, $P_j(t)$ the (ideal) nominal price of the industry j goods, $\alpha > 0$ a labor disutility parameter, $R(t)$ the nominal interest rate, λ the Lagrange mutliplier of the consumer's budget constraint and $Q(t) \equiv e^{-\int_0^t R(s)ds}$ is the price of the time t nominal bond.[5]

From these conditions, we obtain that $Y_j(t) = \log c_j(t) - \log \bar{c}_j = \frac{1}{\epsilon_j}\left[\log\frac{W(t)}{P_j(t)} + \mu_j\right]$ where we use that output is demand determined. Continuing with the analysis of the first order conditions shows that the steady state interest rate is $\bar{R} = \rho + \frac{\dot{M}}{M}$, so that $\bar{R} = \rho$ if the steady state money stock is constant. Moreover we have that $\alpha M(t)R(t) = W(t)$ showing that shocks to the money supply or the interest rate immediately map into nominal wages. Using the definition of $Q(t)$ we have $W(t) = \frac{\alpha}{\lambda}e^{\int_0^t(R(s)-\rho)ds}$ or $W(t) = W(0)\ e^{\int_0^t\left(R(s)-\bar{R}\right)ds}$. Letting $\bar{W} = \lim_{t\to\infty} W(t)$ be the new (after the shock) steady-state wage we write

$$\mathcal{W}(t) \equiv \log\frac{W(t)}{\bar{W}} = -\int_t^\infty \left(R(s)-\bar{R}\right)ds \tag{7}$$

Equation (7) shows that a transitory deviation of the interest rate from the steady-state implies a time varying path of nominal wages. The often studied once-and-for-all shock to the money supply amounts to a shock that immediately triggers a new steady state level of the nominal wage, $\bar{W}$, with no effects on the path of the interest rate ($R(s) = \bar{R}$ for all s). In general a monetary shock is made of two independent components: the permanent effect on nominal wages, and the transitory

[5]The budget constraint is $M(0) + \int_0^\infty Q(t)\left(\tau(t) + W(t)L(t) - R(t)M(t) - \int_0^1\sum_{j=1}^n p_{ji}c_{ji}di\right)dt$, where τ is a lump sum transfer. See Appendix B in Alvarez and Lippi (2014) for a detailed analysis of this model.

wage deviations due to the nominal rate changes.[6]

2.2.2 The firm's price setting problem and aggregate shocks

We consider a second-order approximation of the firm's profit function around the optimal price, so that the firm's period cost is given by

$$F(x, \mathcal{W}(t)) \equiv B_j \Big(x(t) - \mathcal{W}(t)\Big)^2 \tag{8}$$

where $B_j > 0$ is related to the curvature of the profit function for a firm in industry j.

To adjust its price and control the markup the firm must pay the fixed menu cost ψ_j. Alternatively, with a rate ζ_j per unit of time, the firm can adjust the price at no cost (a free adjustment opportunity a la Calvo). This price setting technology, akin to the Calvo-plus model of Nakamura and Steinsson (2010), allows us to span a large class of sticky price models in between the canonical menu-cost model and the Calvo model.[7]

The firm solves the following stopping-time problem

$$\min_{\tau_i, x_i^*} \mathbb{E}\left[\int_0^\infty e^{-\rho t} B_j \Big(x(t) - \mathcal{W}(t)\Big)^2 dt\right] + \sum_{i=1}^{\infty} e^{-\rho \tau_i}\, \mathcal{I}(\tau_i)\, \psi_j \tag{9}$$

where $\rho > 0$ is a discount rate, τ_i a price-reset time, and the indicator function $\mathcal{I}(\tau_i) = 0$ if the stopping time is due to a free-adjustment opportunity. Intuitively, the firm's problem is to control $x(t)$ to track $\mathcal{W}(t)$. The time invariant parameters B_j, ζ_j, ψ_j and σ_j^2 depend on the industry j. Absent aggregate shocks, i.e. $\mathcal{W} = 0$, each firm's gap is affected only by the idiosyncratic productivity shocks. The firm's steady-state policy in industry j consists of a region where control is not exercised if $x \in [\underline{x}_j, \bar{x}_j]$; outside of this region control is exercised and the state is reset to x_j^*. Since the state is driftless the symmetry of the problem implies that $\underline{x}_j = -\bar{x}_j$ and $x_j^* = 0$, i.e. it is optimal for firms to "close the price gap" upon adjustment.

We note that the Calvo-plus setup adopted here allows us to nest a large class of sticky price

[6]The nominal rate changes correspond to a path of money growth, easy to compute given the above equations.

[7]This basic setup, allowing for a random menu cost, can be made more general using a generalized hazard function as in Caballero and Engel (1999, 2007), see Alvarez, Lippi, and Oskolkov (2022) for an extensive analysis of this case.

models in between the canonical menu-cost model and the Calvo model. These models are indexed by a single parameter, the "Calvoness index" $\ell_j \equiv \sqrt{\frac{2\zeta_j}{\sigma_j^2/\bar{x}_j^2}}$, namely the ratio between the number of free adjustments (ζ_j), and the number of adjustments that occur in a canonical menu cost model ($\sigma_j^2/\bar{x}_j^2$). If $\ell_j \to 0$ then the model corresponds to the canonical menu cost problem, while $\ell_j \to \infty$ gives the Calvo model. Both the frequency of price adjustment and the kurtosis depend on Calvoness index. In particular, the kurtosis of the size distribution of price changes is an increasing function of ℓ_j only, ranging between $Kurt = 1$ for $\ell_j = 0$ to $Kurt \to 6$ as $\ell_j \to \infty$.

Modeling aggregate shocks. The class of models we consider posits that monetary shocks affect the firm's marginal costs. In these models nominal wages and the money supply are proportional to each other, so that a positive monetary shock $\delta > 0$ increases the marginal cost of all firms. If the shock is permanent, it permanently increases $\bar{W}$, reducing the markup gaps x of all firms by $-\delta$, see equation (1). At this new level of the money supply, firms charge prices/markups that are too low, thus the output level increases (this is the impact effect). Over time prices will permanently adjust up and output will return to the steady-state level.

In the traditional analysis focusing on permanent monetary shocks, $\mathcal{W}(t) = 0$ for all $t > 0$ since the firm's nominal cost jumps up at time zero and remains constant afterwards. In this case the firm's decision rules are unaffected by the aggregate shock (see proposition 7 in Alvarez and Lippi (2014)) and upon adjustment firms "close the gap". Instead, when the shock involves a whole path $\mathcal{W}(t)$, the firm's decisions are given by three time paths: $\underline{x}_j(t), \bar{x}_j(t)$ and $x_j^*(t)$ for each industry j. These are not stochastic processes, just functions of time. We show next that a time-varying path for $\mathcal{W}(t)$ arises after a transitory shock to the interest rate, as often considered in monetary analyses. In this case the optimal pricing policy at time t is represented by the interval $(\underline{x}_j(t), \bar{x}_j(t))$ so that if $x(t)$ is in this interval the firm in industry j does *not* exercise control, i.e. inaction is optimal. Instead, if $x(t) \notin (\underline{x}_j(t), \bar{x}_j(t))$, the firm immediately changes its price from $x(t^-)$ to $x(t^+) = x_j^*(t)$. The optimal policy $x_j^*(t)$ depends on the future path of $\mathcal{W}(s)$, for $s > t$, and hence the optimal policy upon adjustment is in general different from "closing the gap" ($x_j^*(t) = 0$).

The monetary shock and mapping to the data. To summarize, we consider a monetary shock made of two independent components: a permanent increase of the steady state nominal wages, which we assume equal to δ, and the transitory component given by $\mathcal{W}(t) = \delta\omega(t)$. Below we will focus on the exponential function $\omega(t) \equiv w_0 e^{-\gamma t}$ to parametrize the initial size of the interest rate shock (through ω_0 and equation (7)) and its persistence $1/\gamma$. Note that $\mathcal{W}(0) = \delta\omega_0 = -\int_0^\infty (R(s) - \bar{R})ds$. If $R(t) \equiv \bar{R} + \delta\hat{R}_0 e^{-\gamma t}$, then a given $\hat{R}_0$ implies that $\omega_0 = {-\hat{R}_0}/{\gamma}$. For instance, a 1% increase in the long run nominal wage corresponds to $\delta = 0.01$. To supplement this shock with a 25 basis points reduction of the interest rate with a half life of 1 year we set $\hat{R}_0 = {-1}/{4}$ and $\gamma = 0.69$, which implies $\omega_0 = 0.36$. The left panel of Figure 1 shows an example of a mean reverting interest rate shock, starting with an "expansionary" reduction of the interest rate (equal to 25 basis points) and an exponential decay with a half life of 1 year. The panel also shows the corresponding sequence for $w(t)$. The right panel shows the corresponding response of aggregate prices.

2.2.3 The CIR with a predictable transitory shock

To simplify the notation of this section, we set $\epsilon_j = 1$ and also omit the industry subindex j. We let CIR_0^Y denote the case where the shock consists solely of the permanent component δ, obtained when the $\mathcal{W}(t) = 0$ for all t. We let CIR^Y denote the cumulative output defined in equation (4) for the case allowing for both the permanent and the transitory shock. We show in Appendix A.3 how to solve the impulse response for the general case in which $\mathcal{W}(t) \neq 0$, and use the results to establish the following proposition, for a shock with a transitory component $\mathcal{W}(t) = \delta\,\omega_0 e^{-\gamma t}$ where γ parametrizes the half life of the shock and ω_0 its impact effect on the interest rate. We have the following result:

Proposition 1. Consider $\rho \to 0$ and a transitory shock $\mathcal{W}(t) = \delta\,\omega_0 e^{-\gamma t}$. Let $\tilde{\sigma} \equiv \sigma^2/2$ and

$\ell \equiv \sqrt{\frac{2\zeta}{\sigma^2/\bar{x}^2}}$. The Cumulative impulse response of output is given by

$$CIR^Y = CIR_o^Y + \frac{\omega_0}{\gamma}\left\{\frac{\gamma/\tilde{\sigma}}{(\ell^2+\gamma/\tilde{\sigma})} + \frac{\ell^2}{(1-e^{\ell})^2}\left[\left(e^{2\ell}+1\right)\frac{\operatorname{csch}(\sqrt{\ell^2+\gamma/\tilde{\sigma}})}{\sqrt{\ell^2+\gamma/\tilde{\sigma}}} - \left(2e^{\ell}\right)\frac{\coth(\sqrt{\ell^2+\gamma/\tilde{\sigma}})}{\sqrt{\ell^2+\gamma/\tilde{\sigma}}}\right]\right\} \tag{10}$$

The proposition allows us to explore the robustness of the sufficient statistic result in equation (5). It identifies the key determinants of the deviation from the benchmark analysis of the permanent shock: the degree of Calvoness of the model (ℓ), the persistence of the shock ($\gamma/\tilde{\sigma}$), and the size of the shock on impact ω_0. Figure 2 illustrates some results to quantify the deviation from the benchmark result with respect to the half life of the shock. The left panel of the figure considers a "small shock" similar to the one described in Figure 1 where the interest rate decreases by 25bp on impact.

The vertical axis reports the ratio CIR^Y/CIR_0^Y, namely the ratio of the CIR with the transitory shock relative to the CIR without it. This ratio is 1 if the shock's half life is zero, since in this case there is only the permanent component and there is no deviation between CIR^Y and CIR_0^Y. The ratio also converges to 1 as the shock becomes infinitely persistent (i.e. the transitory component vanishes). The biggest deviations occur for shocks with a half-life of about 1 year (about half the frequency of the price changes, which is set equal to 2 per year in the figure). Even so the maximal deviation is rather limited, below 10% of the prediction of the CIR for the case of the permanent shock. The different curves in the figure refer to different degrees of Calvoness. It appears that the largest deviations occur for the pure menu cost model (kurtosis equal 1), and that the deviations are smaller as the model gets closer to the Calvo model (high kurtosis).

Overall, these deviations are small, in view of the fact that equation (5) predicts deviations of the effects of monetary shocks that are in the range of 600% (as kurtosis varies from 1 to 6). In particular, for intermediate values of kurtosis as measured in the data (around 3, see Section 4.2 and Table 1 in this paper), the maximum value of the deviations is about 3%. Overall, we find that result reassuring about the informativeness of the sufficient statistic result, even in the presence of transitory shocks.

3 An Empirical Test for the Sufficient Statistic Result

This section uses the predictions developed above to derive an empirical test of the theory. We will consider an economy made of several sectors, indexed by j, assuming that firms within a sector are similar, i.e. that they have the same response to a common monetary shock. The thought experiment is to hit this economy with an aggregate monetary shock, and to use the variation in the responses observed across the sectors to test the theory.

The multi-sector set-up outlined above allows us to consider sectors that differ in the variability of the idiosyncratic shocks (σ_j), as well as in the pricing frictions (ψ_j). Equation (5) suggests testing the theory using a linear empirical relation between the product-level CIR of output over a long horizon, and the observed product-level ratios of kurtosis to the frequency of price changes. However, highly disaggregated sectoral output or real consumption series (at a monthly frequency) that match exactly the level of disaggregation and high frequency of observations typical of price data, are usually not available. In particular, in the case of France, there are no available monthly consumption volume data available at the same level of disaggregation as the CPI (we conjecture the same holds for other countries). We thus rely in the following on the cumulated impulse response of *prices* rather than output. One advantage of this strategy is also that both the micro and sectoral sets of variables derive from the same source of micro prices, ensuring consistency.

To obtain this alternative test, let us derive the relation between the cumulated response of output in sector j at horizon T, $CIR_T^{Y_j}$, and the one of the price level at the horizon T, $CIR_T^{P_j} \equiv \int_0^T P^j(t)dt$, following a monetary shock of size δ. To ligthen up notation we assume a permanent shock so that $\log \bar{W} = \delta$ and $\mathcal{W}(t) = 0$ for all t. Using equation (3) and equation (4) we have

$$CIR_T^{Y_j} \equiv \int_0^T Y_j(t)dt = \frac{1}{\epsilon_j} \int_0^T \left(\delta - P^j(t)\right) dt = \frac{1}{\epsilon_j} \left(\delta T - CIR_T^{P_j}\right) \tag{11}$$

where $\frac{\delta}{\epsilon_j} T$ is the cumulated change in nominal output following a permanent increase in money.[8]

[8]Note that when T tends to infinity, as the CIR of output is finite, the CIR of prices diverges. This is an expected property as the price level is permanently higher (or lower).

Replacing $CIR_T^{Y_j}$ by its value in equation (5) we have the following prediction relating the cumulated response of prices and the ratio of the price change distribution for a large T:

$$CIR_T^{P_j} \approx \delta T - \frac{\delta}{6} \frac{Kurt_j}{Freq_j} \tag{12}$$

where the approximation is due to the fact that the theory is based on a second order approximation and that our measurement will use a finite horizon ($T < \infty$).

From this equation, we derive an empirical linear specification linking the product-level CIRs of prices to a monetary shock and the observed product-level ratios of kurtosis over frequency of price changes (in levels). One advantage of this specification (using CIR of prices instead CIR of output) is that the predictions for prices are independent of the sectoral elasticity ϵ_j, which simplifies how the regression coefficient should be interpreted. This provides an additional motivation for focusing on the response of prices rather than output. We will thus estimate, as a baseline, the following linear regression:

$$CIR_T^{P_j} = \alpha + \beta \left(\frac{Kurt_j}{Freq_j} \right) + \nu_j \tag{13}$$

where $\beta = -\delta/6$ is the theory-implied value of the regression coefficients and ν_j is the regression's error term. In our empirical exercises, we have normalized our measure of the monetary policy shock so that $\delta = -1\%$, leading, under a strict interpretation of the model, to the prediction that $\beta = 1/6$. We refer to this regression as the baseline regression, or as a "constrained regression", since the specification imposes that kurtosis and frequency enter the regression as a ratio.[9]

We can further decompose equation (12) to investigate the restriction imposed by the theory on how kurtosis and frequency relate to the CIR. For that, we rely on a first-order Taylor expansion around the sample means $\bar{F}, \bar{K}$, and we get: $CIR_T^{P_j} \approx CIR^{\bar{P}_T} - \frac{\delta}{6} \frac{\bar{K}}{\bar{F}} \frac{Kurt_j}{\bar{K}} + \frac{\delta}{6} \frac{\bar{K}}{\bar{F}} \frac{Freq_j}{\bar{F}}$. From this expression we derive an unconstrained version of the empirical test:

$$CIR_T^{P_j} = \beta_0 + \beta_k \left(\frac{Kurt_j}{\bar{K}} \right) + \beta_f \left(\frac{Freq_j}{\bar{F}} \right) + \nu_j \tag{14}$$

[9] An interesting property of the specification in equation (13) is that, for some type of measurement errors (namely a fraction of price changes being spurious changes, of a small size), the induced multiplicative bias on measured kurtosis and frequency is identical, so these biases do cancel. In other terms the specification is correct even though both kurtosis and frequency are measured with errors. See Supplementary Appendix for details.

The theory suggests that $\beta_k = -\beta_f$, i.e. the slope coefficients of the regressors $\frac{Kurt_j}{K}$ and $\frac{Freq_j}{F}$ are expected to have opposite signs and to be equal in absolute value.

4 Measuring Monetary Shocks and Sectoral Moments

This section discusses the data used in the analysis, and the construction of the empirical statistics needed to test the sufficient statistic result. We use variations across products to test the theory. We rely on the existing cross-product variability in the price adjustment statistics, and on the fact that equation (13) is expected to hold across different sectors.[10] We need to estimate two types of statistics: (i) the cumulative impulse response of prices (CIR^P) computed at the sectoral level, and (ii) the moments of the distribution of price changes for the corresponding products. Section 4.1 and Section 4.2, respectively, present our approach and results in computing those statistics.

Before providing more details on the construction of the objects underlying our test, we stress two important features of our empirical approach. First, we make use of a cross section of moments computed from two micro data sets of prices in France: a first one covering consumer prices and the other one producer prices. Both data sets are relevant for our purpose, and each has distinctive advantages. Consumer prices are observed directly and somewhat less prone to measurement issues (since they can be directly observed in outlets), offer a broader coverage of the economy (goods and services vs. only goods for PPI products) and consumer inflation is used for the definition of the monetary policy target. Producer price data are conceptually closer to the firms' pricing problem studied in standard macro models, and are not affected by sales and temporary promotions.

The second feature is that we identify the monetary shocks by imposing that they have the properties highlighted by the theory (in the spirit of the "sign restriction" approach). In particular, we want a (contractionary) shock to decrease output in the short run, to have a permanent negative effect on the price level, and to have no long-run effect on output. These characteristics are consistent with the theoretical model described above, and are thus desirable to perform a test of the sufficient statistic result. Note that in principle any common shock to the marginal cost of

[10] In the paper we use indifferently the terms "sectors" and "products". For PPI, product and sector classifications fully overlap, whereas for CPI, we will use product specific price indices.

firms could be used to test the theory. Oil price shocks would for instance qualify, but empirically the sectoral dynamics following such a shock is strongly heterogeneous making it hardly useable for a test in a finite sample. On the contrary, an aggregate monetary shock has the desirable features that it will eventually move all nominal prices by the same amount, leaving relative prices unaltered. We exploit this homogeneity property in our long-run identification of the monetary shock. Finally, we stress that another feature of our approach is that the construction of the CIR^P variables does not use the micro data nor the sectoral moments, so there is no reason to expect any bias in favor (or against) the sufficient statistic result.

4.1 Measuring the Sectoral Response to a Monetary Shock

To estimate the CIR^P for a large number of sectors we employ a Factor Augmented VAR (FAVAR), a method developed by Bernanke, Boivin, and Eliasz (2005) and Boivin, Giannoni, and Mihov (2009). We closely follow the approach of Boivin, Giannoni, and Mihov (2009) as they focus on the response of sectoral inflation rates to monetary policy shocks. A brief description is as follows: the FAVAR is a model in which the dynamics of a large number of time series is governed by the evolution of a small number of factors, that are typically – but not necessarily – unobserved and follow a VAR process (see Appendix B for a more detailed description of the FAVAR model).

Formally, the vector of a large number n of time series X_t, called informational time series, are related to the factors F_t by the following equation: $X_t = \Lambda F_t + e_t$, where F_t is a vector of dimensions $K+M$ of respectively unobserved and observed factors, and e_t is a vector n x 1 of error terms with zero mean. Following Boivin, Giannoni, and Mihov (2009) we allow one factor, the interest rate i_t, to be observed, so $F_t \equiv [\tilde{F}_t \; i_t]'$ where the unobservable factors $\tilde{F}_t$ are to be estimated. Notice that the observable factors and the informative time series are two distinct objects that do not have any time series in common. The factors follow a VAR process: $F_t = \Phi(L)F_{t-1} + v_t$ where $\Phi(L)$ is a lag polynomial of finite order and v_t is an error term with zero mean and covariance matrix Q.

We are interested in estimating the response of the disaggregated time series of prices (PPI and CPI) after a monetary shock; in our analysis an exogenous shock to the 3-month Euribor. In a first step, factors are computed from a Principal Component Analysis using the informative time series.

We include three types of "informative time series" in vector X_t (see Appendix B for details): (i) macroeconomic data for France (ii) financial and monetary variables relevant for the euro area (iii) disaggregated series of industrial production indices (IPI), producer price indices (PPI) and consumer price indices (CPI), for France (seasonally adjusted and taken in log differences). In addition, our analysis uses the 3-month Euribor as a measure of the monetary policy variable. This variable is treated as an observable factor, and we filter it following motivations and a procedure that are detailed below. Data are monthly and the sample period is Jan. 2005 to Dec. 2019. From this first step, we extract five principal factors (those with the largest contributions to the overall variance) and we then estimate a VAR model with 12 lags for the 5 factors and the interest rate. From this VAR, we can retrieve the impulse response function (IRF) of all sectoral prices to an aggregate shock. The dynamics of inflation in sector j will, in our FAVAR set-up, governed by:

$$\pi_{jt} = \lambda_j F_t + e_{jt} \tag{15}$$

where λ_j is a vector of loadings, recovered as the relevant row of matrix Λ. From these sectoral IRF, the $CIR_T^{P_j}$ is calculated as the cumulated response of sectoral price levels over a large number of periods (see next section for a discussion).

Identifying Monetary Policy Shocks and the Price Responses. To identify a contractionary monetary shock in our FAVAR model, we use a Cholesky decomposition of the variance-covariance matrix of the VAR innovations. Following a standard timing restriction, the Euribor is ordered as a last variable in the VAR. Notice that, imposing a Cholesky decomposition in this setup does not imply that the IRFs of informative time series cannot respond simultaneously to the monetary shock.

We consider several alternative approaches in identifying monetary policy shocks. In our baseline approach, we impose a "long run neutrality" restriction. Specifically, it is imposed that (i) output comes back to its original level in the long run after a monetary shock and (ii) all sectoral prices have identical responses —equal to that of the average price across sectors— in the long run.[11] Both of these restrictions are consistent with the money neutrality hypothesis. We follow Boivin, Giannoni, and Mihov (2009) to implement the latter restriction in the baseline FAVAR specification.

We also consider an alternative case without "long run neutrality" restriction (a case also considered by Boivin, Giannoni, and Mihov (2009)). In addition, following Gertler and Karadi (2015), we explore a third alternative identification procedure using a High Frequency Identification (HFI) in the VAR set-up. This allows us to deal with simultaneity issues without resorting to a timing assumption (as in the Cholesky approach). In this approach, we use monetary surprises in the euro area computed by Altavilla et al. (2019), relying on market interest rate changes around the times of ECB Governing Council meetings.[12]

In all FAVAR specifications, we normalize the shock, so that the monetary policy shock produces a 1% long-run decrease in the aggregate price level. This normalization assumption (which has no bearings in terms of inference) departs from the usual approach to normalizations imposing that the shock produces an effect on impact on the nominal interest rate. The normalization allows an easier comparison with our theoretical model (where the size of the shock is proportional to the long run response of the price level) and facilitates the interpretation of results relating the CIR^P to the sufficient statistic.

Filtering the Euribor. The theory suggests that a (contractionary) monetary policy shock triggers a transient, and negative, impact on inflation and output. The VAR estimates based on unfiltered interest rate data produce IRFs that are not consistent with these predictions, a feature we relate to the marked downward trend in the nominal interest rate over the sample period (see Figure A.1 — a pattern likely related to the decline in the "natural rate" of interest).[13] Furthermore, the theory also suggests that all the sectors should have a negative IRFs of prices after a contractionary monetary shock.

[11]In practice, we impose these restrictions at the horizon of 8 years. Note this horizon is independent and substantially longer than the one over which we will compute cumulated IRFs (3 years in the baseline).

[12]In a robustness exercise, we also report results using a longer term interest rate -the 2-year German Bond rate- as the policy rate and using the same HFI approach, to account for non–conventional monetary policy shocks.

[13]Identifying well-behaved monetary policy shocks for the euro area is particularly challenging over the sample period, in particular due to the proximity of the effective lower bound on interest rates – see Andrade and Ferroni (2021) and Jarocinski and Karadi (2020) for investigations in the context of information shocks.

In our FAVAR model, we thus filter the interest rate to ensure that the FAVAR model produces a negative and transient response on output and inflation after a contractionary monetary policy shock. To do so, we use a one-sided HP filter, that does not use future data at one point in time, as a hedge against introducing spurious correlations when using the filtered interest rate in the VAR model. Our approach is to use a one-sided HP filter with a parameter λ^{HP} that maximizes the number of PPI and CPI sectors with negative IRFs after 36 months of the shock. Appendix C provides more details on our strategy to select the value for λ^{HP}.[14] Notice that our selection criterion relies on the sign of IRFs after 36 months, not on the CIR which will be used in our regressions. Furthermore, our procedure for selecting the filter parameter makes no use of the microeconomic data or the sectoral moments, so it is not biasing towards finding some relevance of the sufficient statistic results. Our FAVAR estimation procedure is designed to produce a shock that has the same negative price effect in all sectors and can be interpreted as a monetary policy shock.

We select a value of $\lambda^{HP} = 500,000$ for which about 70% of PPI and CPI products have a negative IRF after 36 months (see Appendix C for more details).[15]

VAR Results: IRFs and CIR^P's. Our estimated FAVAR provides theory-consistent results for the responses of aggregate variables to a monetary shock. After a contractionary policy shock the interest rate increases and subsequently decreases, going back to its steady state level in the long run (IRFs are presented in Appendix Figure A.4). Industrial production immediately shrinks after a contractionary monetary policy shock, then gradually recovers. The production price index and the consumption price index both decline following the shock, then recover towards the new steady-state value.

We focus our analysis on the objects used to test the sufficient statistic result, namely the responses of sectoral producer and consumer prices, as derived from the FAVAR. Figure 3 reports the estimated IRFs of production and consumer price series. In each panel, dashed red lines are

[14] Note that the literature does not agree on specific value for the one-sided HP filter, unlike with the standard two-sided HP filter.

[15] As robustness, we have also run the whole empirical exercise including FAVAR and OLS product-level regressions with $\lambda^{HP} = 1M$ and results are qualitatively and quantitatively the same. Results are reported in the Supplementary Appendix Table B.7, Table B.8 and Table B.9.

the IRFs of different sectors partially aggregated, at the 2-digit level for PPI, and 1-digit level for CPI.[16] The thick red line is the average of all the dashed red lines. In both figures, we impose that the long run price response is -1 percent at a long horizon (8 years). The transitory dynamics is however heterogeneous across sectors. Most of them display a through in prices after 1 to 3 years after the shock.

Finally, using the estimated IRFs of the PPI and CPI, we construct the CIR^Ps for each sector/product category, as the sum of the respective IRF from time zero up to a time horizon T. We select a baseline value of $T = 36$ months to compute CIR^Ps (see Table A.1 in Appendix for descriptive statistics on product-specific CIR^Ps) but we will also provide robustness analysis using two different values of T (24 and 48 months).

4.2 Measuring Micro Moments

Consumer Price (CPI) Micro Data. For consumer price micro data, we rely on longitudinal data sets of monthly price quotes collected by the Institut National de la Statistique et des Études Économiques (INSEE) to compute the monthly French CPI (Consumer Price Index). Stacking data sets used in Baudry et al. (2007), Berardi, Gautier, and Le Bihan (2015) and Berardi and Gautier (2016) and extending the data set to September 2019, we obtain a long sample covering a period of about 25 years between August 1994 and September 2019.

The data set contains about 30 million of price quotes, and covers about 60% of the CPI weights.[17] Price changes are computed as log-differences of prices, and we exclude price changes due to sales. To compute price adjustment moments, we have first dropped data collected around VAT changes (i.e. in Aug.-Sept. 1995, Sept.-Oct. 1999, April-May 2000, July-Sept. 2009, Jan.-Feb. 2012 and Jan.-Feb. 2014) and before and after the euro cash changeover (between Aug. 2001 and June 2002). We have also dropped price changes smaller than 0.1% in absolute values, in both data sets, in order to control for possible small price changes due to measurement errors

[16] Our PPI/CPI series are available at the 4-digit and 5-digit levels, and the dashed red lines are constructed as the arithmetic average of estimated IRFs.

[17] Some categories of goods and services are not available in our sample: centrally collected prices, among which car prices and administered prices (e.g. tobacco) or public utility prices (e.g. electricity), as well as other types of products such as fresh food or rents.

(Eichenbaum et al. (2014)).

We compute price adjustment statistics excluding sales, as the model is not able to reproduce price changes due to sales. For identifying sales we rely on an INSEE flag variable that identifies whether a price corresponds to a sale price, either in the form of seasonal sales or temporary promotional discounts.

We identify products at the 5-digit level of the ECOICOP product classification, which is the most disaggregated level for which sectoral price indices are available. For each product, we compute the frequency of price changes as the ratio between the number of price changes (but excluding price changes due to sales) and the total number of prices for this product. We also compute the kurtosis of price changes, as well as other moments of the price change distribution (such as average price changes, the standard deviation of price changes and the skewness of price change distribution), at the product level. Overall, our baseline data set contains price adjustment moments for 223 different "ECOICOP-5" CPI products.

Measurement of kurtosis is notoriously a challenging issue, as large values of price changes, and outliers, can have an important impact on kurtosis. Very large kurtosis values tend to be obtained when not correcting for measurement errors.[18] In our baseline, we drop from the calculations price changes larger than 25% in absolute values, which corresponds to about five percent of all price changes. As robustness, we provide results with alternative values for the thresholds used to defining for outliers and address measurement errors concern (for very large or very small price changes in absolute values). Drawing on Alvarez, Lippi, and Oskolkov (2022), we also provide results using a measure of kurtosis including a correction for unobserved heterogeneity. Alternative kurtosis measures are highly correlated across products.

Producer price (PPI) Micro Data. We rely on micro price data collected by INSEE to construct the French Producer Price Index, this data set is the same as the one used in Gautier (2008) where further details are available. Reported prices must be observed at the "factory gate", ex-

[18]Note however that excluding sales by itself does not decrease the degree of kurtosis, see for instance Gautier and Le Bihan (2022).

cluding transport and commercialization costs, or invoiced VAT.[19] Our sample contains more than 1.5 million price reports between January 1994 and June 2005. Overall, more than 90% of the price quotes used to compute the French PPI are available. The PPI covers all products manufactured and sold in France by industrial firms, which includes sections C (Mining and quarrying), D (Manufacturing) and E (Electricity, gas and water supply) of NACE Rev 2 classification. As for CPI, price changes are computed as log-differences of prices.

For each NACE 4-digit sector, we compute both the frequency of price changes and the kurtosis of non-zero price changes, as well as other moments of the price change distribution. Unlike with CPI, large price changes are much less frequent (reflecting that sales or temporary promotions are absent in the business-to-business context of producer prices) and only 2% of all price changes are larger than 22% in absolute value. To measure kurtosis, we drop price changes larger than 15% in absolute values (which correspond to less than 5% of all price changes) and we test the robustness of our results to this definition of price change outliers. We restrict to the subsample of sectors for which an aggregate sectoral price index is available from the statistical office, so as to match micro moments with time-series macro evidence in our subsequent analysis. This results in a baseline sample containing 118 sectors.

Basic statistics for the micro data underlying both the CPI and the PPI are presented in Table 1. Consumer prices are more rigid than producer prices, with average frequencies of price changes of 10.6 percent and 19 percent respectively. The distribution of price changes has fat-tails for both data sets, with a virtually identical value of the unweighted average kurtosis of 5.0 in both data sets. One main important takeaway is there is some cross-sectoral dispersion in frequency and kurtosis of price changes, for both consumer prices and producer prices - as apparent from the interquartile ranges or standard deviations.[20] The frequency of price changes however seems to show relatively more cross-sectoral variability than the kurtosis of price changes. While alternative corrections for measurement error and unobserved heterogeneity do change the average value of kurtosis, they do not substantially affect the degree of cross-product heterogeneity however.

Cross-sectoral characteristics of both our CPI and PPI data sets are consistent with available

[19]Contrary to CPI prices, there is no flag for temporary promotions or sales. We assume, consistent with Nakamura and Steinsson (2008), that there are no sales in producer prices.

[20]Figure B.1a and Figure B.1b in the Supplementary Appendix plot the full distribution of moments.

international evidence. As regards consumer price data, Berardi, Gautier, and Le Bihan (2015) using the same data, provide a detailed comparison of CPI data moments in France with those in the United States, based on detailed moments reported by Nakamura and Steinsson (2008). They conclude that patterns are quite similar, whenever sales-related price changes are disregarded (as the pattern of sales is however much more prevalent in the United States). Regarding producer prices, Vermeulen et al. (2012) provide a comparison of the patterns of price setting in the United States and 6 euro-area countries, including France - relying for that particular country on the same data set as we use. They conclude patterns of producer price rigidities are very similar - albeit the size of price changes is typically larger in the United States than in Europe. The above-mentioned international evidence mainly focuses on the frequency of price changes, as well as on the first two moments of the distribution of price changes. Evidence is scarcer on kurtosis. For US PPI data, Hong et al. (2020) report and average kurtosis of 4.9. With consumer price data, Cavallo (2018) report a median kurtosis of 4.8 in a large sample of countries based on "scraped" data. These values, all obtained after correcting for measurement errors in the same spirit as we do, are thus much in line with our baseline values.

5 Testing the Theory: Results

This section presents the results of the empirical tests developed in Section 3 using as inputs the product-level CIR^P (as measured in Section 4.1), and product-level moments of price adjustments (as measured in Section 4.2).

5.1 Estimates of the Baseline Empirical Specification

This section presents our baseline estimation results. As detailed in Section 3, the theory predicts that, in case of a contractionary monetary policy shock, the coefficient associated with $Kurt/Freq$ ratio should be positive in the regression for regression for CIR^P. Indeed, a contractionary monetary policy shock induces negative CIR^Y and CIR^P for all products. Consistent with the theoretical prediction, products with smaller $Kurt/Freq$ ratios are expected to experience a negative

CIR^Y with smaller absolute value and conversely a more negative CIR^P, resulting in a positive coefficient associated with the $Kurt/Freq$ ratio in the cross-section regression for CIR^P. A smaller $Kurt/Freq$ ratio can reflect either more frequent price adjustments, more price selection or both, implying smaller (absolute) real effects of a monetary policy shock and larger (absolute) cumulated price response.

Table 2 reports results for equation (13), the baseline "constrained" regressions for horizon T equal to 36 months.[21] Regressions for the CIR^P of PPI products are presented in Panel A while those for the CIR^P of CPI products are presented in Panel B. In each panel, we report results for the three specifications for the identification of the monetary policy shock: (i) the baseline one, with Cholesky identification and long run restriction on relative prices (columns 1 and 2); (ii) identification using Cholesky but not imposing any restriction on the long-run effect on relative prices (columns 3 and 4); (iii) another alternative where identification relies on High Frequency Identification (HFI) (columns 5 and 6). For each type of monetary shock, we run regressions as in equation (13) without including any product "fixed-effects" (columns 1, 3 and 5). Instead in regressions reported in columns 2, 4 and 6, and labeled as including "fixed-effects", we include dummy variables for 2-digit level sectors for both CPI and PPI products. There are 38 such broad 2-digit sectors in our sample for the CPI, and 24 in the case of the PPI.[22] We are agnostic on whether the "fixed effect" case, or the no "fixed effect" one, is the most relevant specification. If variability in the kurtosis-to-frequency ratio is mainly across broad sectors, then introducing fixed effects will act as a confounding factor and may obscure the results in a finite sample of data. By contrast, beside providing a hedge against spurious correlation, including fixed effects is relevant if the relation between CIR^P and the pricing-moments holds within broad sectors. Both specifications inform us on the sources of product variability that help to identify the relation between CIR^P and the cross sectional moments: broad sector differences versus within-sector variability.

[21] Figure B.2 in Supplementary Appendix plots product-level CIR^P against the product-level ratio $Kurt/Freq$, for the different FAVAR specifications and for both PPI products and CPI products. They illustrate a positive relationship in the cross section of products between the value of CIR^P and the value of the ratio $Kurt/Freq$ for most specifications.

[22] In the Appendix, as robustness, we also report results using dummy variables for more aggregate sectors (6 for PPI and 12 for CPI).

For producer prices (Panel A), the estimated slope coefficient associated with the $Kurt/Freq$ ratio turns out to be positive and statistically significant in most cases. Adding "fixed effects" sectoral dummy variables weakens the significance of the estimated parameters, but the results are qualitatively and —for most of coefficients— quantitatively the same as in our baseline regressions. These results are consistent with the theoretical framework. Coefficients obtained in the case without the long-run restriction (cols 3 and 4) are significant and with expected signs but are larger than in the baseline case (cols 1 and 2), presumably reflecting a larger degree of variability of the sectoral CIR^P (see Table A.1 in Appendix).[23]

For consumer prices, the results (Panel B of Table 2) are mixed. When sectoral fixed effects are not included, the $Kurt/Freq$ ratio is small and not significant. When sectoral fixed effects are included, the coefficients are positive and significant in the three specifications. These results suggest that the relationship between CIR^P and the pricing moments is driven mainly by within-sector variability rather than broad sector differences. For CPI, the incidence of sales could be of particular importance to explain why the relationship does not hold when looking at broad differences across sectors. The extent of sales could indeed affect price adjustment moments even if we have excluded price changes due to sales in the calculation of these moments. In particular, if a large majority of price adjustments are due to sales or promotions in one sector, the pricing moments excluding these changes might be not very representative of the typical price changes. To explore this, we carry alternative estimation exercises removing all food, clothing/footwear and furnishings goods, as within these broad sectors, most products are largely affected by seasonal sales and replacements.[24] In another set of additional estimates, we exclude CPI products for which more than 10 percent of all price changes due to sales (this fraction corresponds to the median value among all CPI products). Results are reported in Table 3. In all specifications, the coefficient associated with the $Kurt/Freq$ ratio is close to the one obtained in our baseline case including sectoral fixed effects. In all except one specifications, we find that this coefficient is significant. Overall, these results suggest that the sufficient statistic prediction better holds for

[23]The estimated values relying on the FAVAR with long-run restrictions are more consistent with our theoretical set-up, while the FAVAR without these restrictions puts less constraint on the data but makes the size of the coefficients more difficult to relate to the exact predictions of the theory.

[24]These products correspond to COICOP 01.1, 03 and 05 in the product classifications.

CPI products less affected by sales.

Finally, the model provides not only predictions on the sign of the coefficients of the regressions, but also on the magnitude of the coefficients. In the constrained version of the model, β is predicted to be equal to $-\frac{\delta}{6}$ which is $1/6 \approx 0.167$ since we use a normalized the shock $\delta = -1$. The order of magnitudes of estimates for PPI in Table 2 in the case of the specifications with long run restrictions, are broadly in line with the predictions. The last row of each panel in Table 2 reports the results of a more formal test for the β coefficient and we cannot reject that the value of the β coefficient is consistent with model's predictions.[25] For CPI, however, the hypothesis that the value of the β coefficient is consistent with model's predictions can be rejected for most specifications. When we consider products less affected by sales (Table 3), for which the β coefficient are in more cases significant, we cannot reject that the coefficient is consistent with the theoretical value in the specification without long run restriction - albeit the result is less favorable to theory for other specifications.

5.2 Estimates of the "Unconstrained" Empirical Specification

To further investigate the relevance of both the kurtosis and the frequency of price adjustments in explaining the propagation of monetary shocks, we report in Table 4 the estimate for equation (14), an "unconstrained" version of the regression that allows for a potentially different effect of frequency and kurtosis.

For PPI products (Panel A), the estimates are consistent with the theoretical predictions in all specifications. First, after a contractionary shock, if prices are more flexible in a given sector (i.e. larger frequency), prices will decline faster and the product-level CIR^P will be more negative. This will induce a negative relationship between the frequency and CIR^P. Second, a smaller kurtosis in a given sector (i.e. a larger selection effect) is associated with a more negative reaction of prices after a contractionary shock, resulting in a positive coefficient in the cross-section regression between CIR^P and kurtosis. When we do not include sectoral fixed effects, coefficients associated with frequency but also kurtosis are all significant at 5% or 10% levels. In the specification with

[25] Table A.2 in Appendix D reports more results on formal tests showing that for PPI we cannot reject that the constant is also equal to -36 as predicted by the theory ($-\delta T = -36$).

sectoral fixed effects, the sign and the size of coefficients remain quite similar but they are not significant any more for both frequency and kurtosis. Our interpretation is the addition of sectoral fixed effects substantially reduces the source of cross-sectional variation and so, lowers the precision of the estimates.

For CPI products (Panel B), we also find – in all cases – a negative and significant relationship across sectors between frequency and the CIR^P, and that the slope coefficient associated with kurtosis is positive. Coefficients associated with frequency are statistically significant in all specifications. Coefficients associated with kurtosis are significant in the case with Cholesky identification and long-run restrictions, and in the IV specification, but not in the case with Cholesky identification and no long-run restriction. When considering CPI products which are less affected by sales (Table 3), in all of specifications the frequency is significantly and negatively correlated with CIR^P. In half of specifications the kurtosis is significantly and positively correlated with CIR^P.

Finally, the last row of each Panel reports a formal test for equality in absolute values of the hypothesis: $\beta_f = -\beta_k$, a property predicted by the theory. For PPI, in all regressions, we cannot reject that slope coefficients associated with frequency and kurtosis are equal in absolute value. For CPI, this is only the case in the Cholesky and IV specifications with long-run restrictions and when sectoral fixed effects are not included (cols 1 and 5).[26]

As a complement to these regressions, note that frequency and kurtosis are significant and have the expected sign in regressions in which they are in turn introduced alone as regressors (see Table A.5 in the Appendix). This broadly reflects the weak correlation in our sample between frequency and kurtosis. However, such a test is not a relevant test of the sufficient statistic in our view.

[26] Table A.2 in the Appendix reports p-values of formal Fisher tests from the estimated parameters. For PPI products, we find that in all specifications, results are fully in line with predictions on the amplitude of the coefficients and we cannot formally reject that the size of coefficients are consistent with model's predictions. It is less the case for CPI products.

5.3 "Placebo" Tests

While the above results are consistent with the "sufficient statistic" property, a sufficient statistic property predicts something broader: it implies that the effect of a monetary shock should be related to the ratio "kurtosis over frequency" but it also implies that other moments of the price distribution should not matter in this relationship. To test this prediction, we run a regression in which we add to our baseline regressors three additional moments of the price change distribution computed at the product level: the average size of (non-zero) price changes, the standard deviation and the skewness of price adjustments. This exercise can be considered as a "placebo" test of our baseline regressions, testing that our main result is not driven by correlations between frequency or kurtosis and other moments of the price change distribution.

Table 5 provides results for this specification. For PPI products (Panel A), the coefficients associated with the ratio of kurtosis over frequency are highly similar to the ones obtained in the baseline case (Table 2). They are much less precisely estimated however, and they remain significant at 5% level only in two specifications without sectoral fixed effects. Importantly, neither the average size of price changes, nor the standard deviation of prices changes, nor the skewness of price changes, do have statistically significant effects in any of the six specifications. These two results are consistent with the theoretical prediction. We have in addition estimated an unconstrained version of the "placebo" regression (results are in Table A.3 in the Appendix). Results for PPI products are broadly robust, although the degree of significance decreases, presumably owing to multi-colinearity.

For CPI products (Panel B), results are, as with the baseline specification, more mixed. The coefficient on $Kurt/Freq$ is positive and significant in only two cases and several coefficients associated with the "placebo" moments are significant (in 8 cases out of 18). Results are also quite mixed when looking at the unconstrained version of the "placebo" regressions (see Table A.3 in the Appendix). We have run the same placebo regressions for CPI products less affected by sales (Table A.4 in the Appendix). In that case, the coefficient associated with $Kurt/Freq$ ratio remains positive and significant in all specifications and only three coefficients (over 18) associated with the "placebo" moments are significant at 10% level.

As alternative "placebo" tests, we have also considered introducing other covariates that may be confounding factors. Candidates are average inflation, production volatility (available for PPI only), or the degree of "upstreamness" in the production chain (as captured by dummies for broad sectors). Results are reported in Table A.13 and Table A.14 in the Appendix. Overall, results are unaffected: the $Kurt/Freq$ ratio remains significant. One qualification, though, is that the variable average inflation turns out to be significant in some cases.

6 Robustness Analysis

This section explores the robustness of our findings with respect to several dimensions: (i) the time horizon of the CIR; (ii) the measurement of kurtosis; (iii) the exclusion of products with a large drift in prices; (iv) the use of long-term bond as a policy indicator (related to the zero lower bound on interest rates and unconventional policies); (v) using moments of price durations as an alternative sufficient statistics; (vi) using the CIR of output as a dependent variable.[27]

6.1 The Time Horizon of the CIR

In our baseline results, the CIR^P is computed cumulating price deviation for 36 months after the shock. We have carried out our various estimations using time horizons of $T = 24$ months and $T = 48$ months and results are reported in the Appendix in Table A.6, Table A.7, Table A.8 and Table A.9. For the 48-month time horizon, the slope coefficients associated with the ratio $Kurt/Freq$ are almost identical as the ones obtained for the 36-month horizon. When we use CIR^P calculated over a 24-month horizon they are lower but still close. As expected, estimates of the intercepts vary with the time horizon. In all regressions, results are quantitatively and qualitatively close to the baseline results.

[27] We have also performed robustness analysis to investigate: vii) the influence of product with outliers for CIR^P, or of frequency of price changes, kurtosis or the ratio $Kurt/Freq$, and viii) the case excluding CPI from the FAVAR (hence from the full analysis). Results were largely unaffected and for brevity these cases are discussed in the Supplementary Appendix.

6.2 The Measurement of Kurtosis

The measurement of kurtosis is known to be severely affected by unobserved heterogeneity. We run robustness regressions using a measure of kurtosis, based on Alvarez, Lippi, and Oskolkov (2022), that takes into account product-level unobserved heterogeneity. Results reported in the Appendix Table A.10 are very much in line with the ones in our baseline regressions. For PPI, the coefficient associated with the $Kurt/Freq$ ratio is positive, and significant in all specifications, whereas for CPI the estimated coefficients are not statistically different from 0. In the unconstrained regression, results are also qualitatively and quantitatively similar to the ones obtained in the baseline regressions.

We also investigate the role of very large or very small price changes (in absolute values) for the measurement of kurtosis. In the baseline regressions, we have used kurtosis measures calculated on the sample of price changes smaller in absolute value than 15% for PPI price changes and than 25% for CPI price changes (i.e. 5% of all price changes in both cases) and we have excluded price changes below 0.1% in both cases. We here test the robustness of our results to modifying the thresholds defining extreme price changes. In a first exercise, we investigate the role of large price changes and we set the thresholds defining extreme values to 25% for PPI price changes and 35% for CPI price changes (i.e. about 2% of all price changes). In a second exercise, we set the threshold for small price changes to 0.5% (which corresponds to about 5% of all price changes).[28] The results overall remain in line with the baseline results (see Table B.10 and Table B.11 in the Supplementary Appendix). Standard errors of coefficients are however higher, lowering the significance of the estimated coefficients, in particular for large producer price changes.

6.3 Using a Long-Term Yield as Policy Indicator

In this robustness, we alter the policy rate used in the FAVAR estimation where the shock is identified using an external instrument approach. The main motivation is that over the last part of our sample the short-run policy rate was arguably constrained by the proximity of the effective

[28] We have also run similar exercises with other definitions of small and large price changes and conclusions are very similar.

lower bound for interest rates, and the ECB engaged in unconventional monetary policies intended to influence long-term interest rates.[29] We use the 2-year German sovereign bond rate, a relevant risk-free long-term interest rate, instead of the 3-month Euribor rate.[30]

Results relating sectoral CIR from this FAVAR model and the sufficient statistic, for PPI products, are in line with the baseline (see Table A.11 in the Appendix). The coefficient associated with the $Kurt/Freq$ ratio is positive and significantly different from 0 (and we cannot even reject the coefficient being equal to the predicted value of 1/6). In the unconstrained specification, the estimated parameters associated the frequency and kurtosis are very close to the ones obtained in the baseline case. For CPI products, the coefficient associated with the $Kurt/Freq$ ratio is positive and significant but small in the case without fixed effects. In the unconstrained version of the model, frequency is negative and significant as in the baseline but the parameter associated with the kurtosis is not significant any more.

6.4 Removing products with sizeable drifts in price levels

The theoretical predictions of the model are derived under the assumption of low inflation. While this assumption is clearly fulfilled for the aggregate inflation rate in France over our sample period, a concern is that for some specific sectors it may not be the case. Table 1 provides some statistics on the average product-specific inflation rates in absolute values. Product-level inflation rates (taken in absolute value) are typically small as well: average and median inflation rates are about 1.5% per year whereas the third quartiles of inflation distribution are around 2%. In this robustness exercise, we remove all products for which we observe a "non-small" average inflation rate (in absolute values). In practice, we define small inflation rates as products with an average annual inflation lower than 5% in absolute values.[31] For PPI products, only two products are removed, whereas for CPI, 9 products are removed. For both PPI and CPI, results are reported in Appendix

[29] Note however that the policy rate was negative from 2014, and statements by the ECB indicate that the lower bound was not actually reached afterwards.

[30] Jarocinski and Karadi (2020) use the 1-year and 2-year German bond as a policy variable in their analysis of ECB monetary policy.

[31] Gagnon (2009), Nakamura et al. (2018) or Alvarez et al. (2019) for evidence on price rigidity in higher inflation environments, they tend to show that when inflation is below 5%, patterns of price rigidity (in particular, frequency of price changes) are rather unchanged.

Table A.12 and they are very consistent with the ones obtained in the baseline regressions.[32]

6.5 Using the CIR of output

Originally, the theoretical results were developed using CIR^Y, but deriving the predictions for CIR^P is straightforward as shown above. We focused our empirical analysis on CIR^P for two reasons. First, product-level measures of output are only available at an infra-annual frequency for producer goods, and not for consumer goods. Second, the sufficient statistic prediction derived for CIR^Y contains a "nuisance parameter", the industry-specific elasticity (ϵ_j), which is not the case for prices where the prediction simply links CIR^P to the kurtosis over frequency ratio. This extra parameter in the prediction for output could blur the quantitative interpretation of the estimated coefficients and might also complicate the estimation of the correlation between CIR^P and the kurtosis-over-frequency ratio.

However, as a robustness exercise, we have performed estimations for output in the case of PPI, using the sectoral Industrial Production Index as product-specific output variable. The shock is normalized the same way as for producer prices.[33] Results of regressions using the CIR^Y's as dependent variables are reported in Table A.15 of the Appendix.[34] Note that the $Kurt/Freq$ ratio is now expected to have a negative sign, opposite to the case of CIR^P. The results turn out to be mixed and generally weaker than using CIR^P. In all cases (whether with "long-run restriction" or not, and with fixed effects or not) the $Kurt/Freq$ has the expected sign. However, it is not significant, reflecting very imprecise estimates (in particular with fixed effects). Our interpretation is that the weaker results are consistent, and in fact to be expected, in the presence of heterogeneity in the industry-specific income elasticity (ϵ_j).

6.6 Using moments from the distribution of price durations

Another robustness test consists in using moments of price durations as a substitute for the candidate sufficient statistic $Kurt/Freq$. Indeed, several authors have exploited the idea that the

[32]Using a threshold at 4% for defining 'small' vs 'large' inflation rates leads to similar results.

[33]The number of products is larger than in the case of prices because more product-level IPIs (than PPIs) are available over a long-time dimension.

[34]Further results are provided in a Supplementary Appendix Table B.4, Table B.5.

distribution of the durations of price-spells is informative about monetary non-neutrality, see e.g. Carvalho and Schwartzman (2015) and Baley and Blanco (2021). As shown by Alvarez, Lippi, and Paciello (2016) (proposition 2), the distribution of durations provides an alternative formula to compute the CIR, involving two moments: the average price-spell duration and the squared coefficient of variation of durations. The former is obviously related to the average frequency of price changes appearing in equation (5), the latter is a stand in for the kurtosis. Indeed, if the distribution of the idiosyncratic shocks is normally distributed the two formulas are equivalent.

Regression results involving these spell duration moments are reported in the Appendix (Table A.16). The specification suggested by Alvarez, Lippi, and Paciello (2016), where duration and the coefficient of variation (CV) enter the regression multiplicatively, appears significant with the expected sign, both for the PPI as well as for the CPI sample. As mentioned above this result is consistent with the ones shown above using the moments from the distribution of price changes. The regressions where the frequency and the CV are entered as separate regressors also show the correct sign but the statistical significance of the CV regressor is weaker. We note that in several models, such as the ones discussed in Section 2, there is a tight link between the distribution of durations and the distribution of price changes. The two distributions encode the same information (see Appendix E in Alvarez, Lippi, and Oskolkov (2022) for a formal analysis of this equivalence). Under the null hypothesis that the model is the data generating process the two tests are equivalent. Differences in the statistical significance of the regressions might be due to differences in the quality of the data (measurement errors in durations vs size of price changes) or reflect deviations from the assumed normal distribution for the firm's idiosyncratic shocks.

7 Conclusion

In a broad class of sticky price models the non-neutrality of nominal shocks is captured by a simple sufficient statistic: the ratio of the kurtosis of the size-distribution of price changes over the frequency of price changes. This paper tested this theoretical prediction using sectoral and microeconomic data for France both for PPI and CPI products. Our test followed three steps. We first measured the effects of monetary shocks using a Factor Augmented VAR across a number of industries. Second, we measured the candidate sufficient statistics using micro data for the same industries. Third, we checked whether the sufficient statistic displays a systematic relation with the non-neutrality across industries, as the theory suggests.

We found substantial support for the theoretical predictions, particularly so in the PPI data. The estimated industry non-neutrality correlates with the kurtosis and the frequency consistently with the predictions of the theory. Several robustness tests are investigated and the results appear solid. The support for the theoretical predictions is weaker on the CPI data. This might possibly be due to seasonal sales (or price plans). Such features, prevalent in the CPI, violate the assumption under which the sufficient-statistic result is derived. Another possible confounding factor is the presence of learning (price discovery), a feature shown by Baley and Blanco (2019) to weaken the power of the sufficient statistic, and which likely is more prevalent in the CPI.

Avenues for future research include considering the sectoral data of other countries or extending the analysis to set-ups that feature a non-negligible drift, like economies with high inflation or the investment problem studied by Baley and Blanco (2021). Another interesting possibility is the use of granular data for additional tests of the theory. A recent test of the sufficient statistic was developed by Gautier, Marx, and Vertier (2021) for the gasoline industry, using granular data on prices measured at gas stations. The data provide an ideal testing ground for the theory, in spite of the fact that they are not representative of a whole economy. The results provide very strong support for the sufficient statistic predictions. Other granular datasets may allow one to test the predictions of the theory and possible explore the state dependence of aggregate shocks, a hallmark of lumpy-adjustment models, as considered by Caballero, Engel, and Haltiwanger (1997) and Caplin and Leahy (1997). Finally, the tractable model we used for the analysis of a mean reverting interest rate shock could be applied to other problems where nominal shocks feature a predictable transitory component, such as forward guidance shocks or sticky wages.

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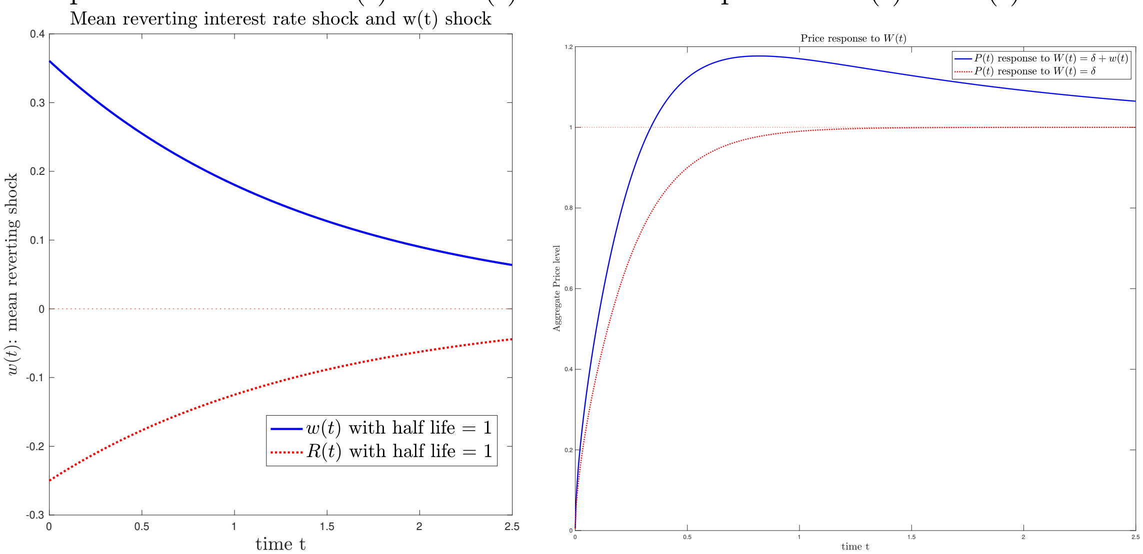
Figures and Tables

Figure 1: Mean reverting interest rate shock

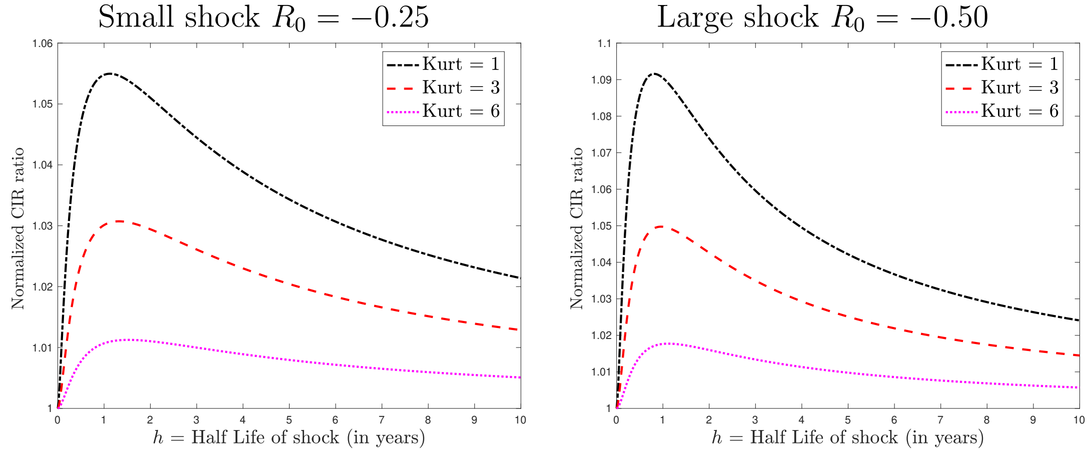
The path for the shock $R(t)$ and $w(t)$ — Response of $P(t)$ to $W(t)$



Note: the nominal shock equals $W(t) = \delta\,(1 + \omega(t))$. The left panel plots the transitory component $\omega(t) \equiv w_0 e^{-\gamma t}$, and the associated nominal interest rate shock $R(t) - \bar{R} = \delta \hat{R}_0 e^{-\gamma t}$ for two values of the initial shock R_0. We set $\delta = 0.01$ and consider a -25 basis points shock to the interest rate with a half-life of 1 year corresponds to $\hat{R}_0 = -1/4$ and $\gamma = 0.69$. By equation (7) then $\omega_0 = 0.36$. The right panel shows the aggregate price response to the $W(t)$ shock (thick line) and to the once and for all shock ($\omega(t) = 0$, dotted line).

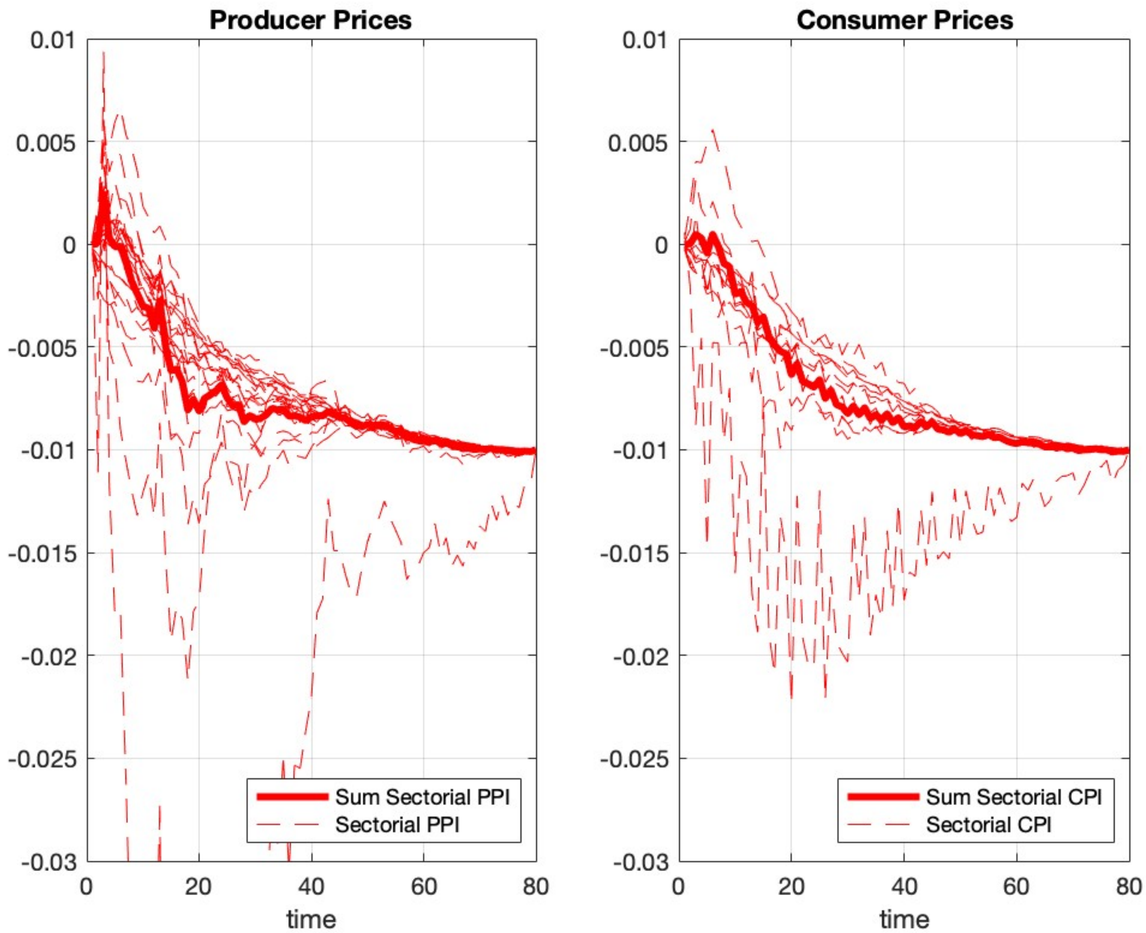
Figure 2: Normalized CIR^Y as a function of the shock's duration

Small shock $R_0 = -0.25$ — Large shock $R_0 = -0.50$



Note: the figure plots the Cumulative impulse response for different degrees of the shock persistence (half-life). Each panel considers three models indexed by the degree of kurtosis (a function of ℓ). The computation uses equation (10).

Figure 3: Sectoral Responses of PPI and CPI to a Contractionary Monetary Shock



Note: this figure plots the impulse response function of product-level prices (measured in log points in deviation from the "steady state" (y-axis)), the left panel corresponds to the sectoral IRFs of PPI products and the right panel corresponds to sectoral IRFs of CPI products. All product-level IRFs are computed at a disaggregate product level; for CPI, the level of disaggregation is 5 digit-level of the ECOICOP classification (ie. '01.1.1.1') whereas for PPI, the product level is the 4-digit level of the NACE rev2 classification of sectors. Each dashed red line corresponds to sectoral IRFs computed at a 2-digit product level (ie. as the simple average over the most disaggregated product level IRFs used then in our OLS regressions), thick red line plots the average IRF computed over all disaggregated product-level IRFs.

Table 1: Micro Moments of Price Adjustments: Descriptive Statistics

	Nb products	Mean	Q1	Q2	Q3	SD
Panel A: Frequency of price changes						
CPI	223	0.106	0.039	0.088	0.143	0.104
PPI	118	0.190	0.086	0.123	0.185	0.208
Panel B: Kurtosis of non-zero price changes - with robustness						
CPI - baseline	223	5.039	3.355	4.434	5.652	2.952
PPI - baseline	118	5.068	3.927	4.615	5.857	1.851
CPI - outlier $\|\Delta p\| < 0.5\%$	223	4.616	3.559	4.281	5.166	1.738
PPI - outlier $\|\Delta p\| < 0.5\%$	118	4.777	3.183	4.220	5.411	2.821
CPI - outlier $\|\Delta p\| > 35\%$	223	6.273	3.880	5.471	7.207	4.316
PPI - outlier $\|\Delta p\| > 25\%$	118	7.805	5.532	6.956	9.042	3.952
CPI - hetero (S=5)	223	3.424	2.227	3.194	3.834	2.013
PPI - hetero (S=5)	118	3.917	2.638	3.435	4.497	2.036
Panel C: Mean of non-zero price changes (percent)						
CPI	223	1.219	0.294	0.947	2.074	2.124
PPI	118	0.793	0.204	0.722	1.405	0.906
Panel D: Standard deviation of non-zero price changes (percent)						
CPI	223	7.587	6.018	7.298	9.251	2.307
PPI	118	4.149	3.606	4.134	4.674	0.872
Panel E: Skewness of non-zero price changes						
CPI	223	-0.261	-0.419	-0.250	-0.098	0.367
PPI	118	-0.274	-0.559	-0.275	0.028	0.444
Panel F: Average inflation (in percent, absolute values)						
CPI	223	1.883	0.663	1.531	2.368	2.123
PPI	118	1.556	0.903	1.327	1.984	1.111

Note: Calculations on CPI micro data are made over the period 1994-2019 (30 million of monthly price quotes). Prices of rents, cars, fresh food products, electricity and clothing goods are non-available or excluded. Price changes due to sales and promotions are excluded (using the INSEE flag). VAT change and euro–cash changeover periods are excluded as well. Calculation on PPI data are made over the period 1994-2005. We here report some descriptive statistics of the distribution of product-specific moments of price rigidity for PPI and CPI products (statistics are unweighted). 'Frequency' reports the ratio between the number of price changes and the total number of prices. 'Mean', 'Standard deviation', 'Skewness' and 'Kurtosis' are calculated on the distribution of non-zero log price changes, expressed in percentages. In our baseline calculations, we have excluded all price changes below than 0.1% in absolute values and larger than 25% in absolute values for CPI price changes and 15% for PPI price changes. Panel F provides statistics on the average product-specific inflation in absolute values over the period 2005-2019. "Hetero (S=5)" refers to the correction for heteroskedasticity presented in the Appendix.

Table 2: Baseline OLS Regression Results : "Constrained" Specification - 36-month horizon

Identification	Cholesky		Cholesky		High-Freq. IV	
Long-run Restriction	Yes		No		Yes	
Product FE	No	Yes	No	Yes	No	Yes
PANEL A: PRODUCER PRICES						
Kurt/Freq	0.206***	0.116**	0.543**	0.231	0.189***	0.135**
	(0.0744)	(0.0544)	(0.247)	(0.175)	(0.0646)	(0.0555)
Constant	-27.93***	-17.85***	-42.64***	-34.19***	-34.08***	-27.29***
	(4.563)	(3.132)	(15.57)	(6.979)	(3.834)	(6.984)
Observations	118	118	118	118	118	118
R^2	0.095	0.534	0.058	0.468	0.110	0.452
P-val $\beta = 1/6$	0.598	0.358	0.129	0.713	0.736	0.568
PANEL B: CONSUMER PRICES						
Kurt/Freq	-0.00221	0.0416***	0.0275	0.0931**	0.0125	0.0328***
	(0.0150)	(0.0151)	(0.0397)	(0.0434)	(0.0107)	(0.0118)
Constant	-16.42***	-11.74***	-19.11***	-20.00***	-23.78***	-21.61***
	(1.958)	(1.190)	(5.763)	(4.438)	(1.582)	(0.768)
Observations	223	223	223	223	223	223
R^2	0.000	0.439	0.002	0.334	0.006	0.649
P-val $\beta = 1/6$	0.000	0.000	0.001	0.092	0.000	0.000

Note: this table reports results of OLS regressions (equation (13)) where the dependent variable is the product-specific $CIR_T^{P_j}$ (calculated for the horizon T=36 months, and expressed in %) and the right-hand-side variable is the ratio $Kurt/freq$. Each observation corresponds to a disaggregate CPI or PPI product. For CPI, the level of disaggregation is 5 digit-level of the ECOICOP classification (ie. '01.1.1.1') whereas for PPI, the product level is the 4-digit level of the NACE rev2 classification of sectors. PPI covers the manufacturing sectors whereas CPI covers about 60% of the whole French CPI (main products excluded are rents, cars, utilities like electricity). Product fixed effects are defined at the 2-digit level for both CPI and PPI products (i.e. 38 product fixed effects for the CPI, and 24 in the case of the PPI). Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 3: Regression Results: Role of sales - Consumer Prices

	Case 1: Excluding food, clothing/footwear, furnishings			*Case 2: Products with % of sales prices below the median*		
Identification	Cholesky	Cholesky	High-Freq. IV	Cholesky	Cholesky	High-Freq. IV
Long-run Restriction	Yes	No	Yes	Yes	No	Yes
PANEL A: Constrained model						
Kurt/Freq	0.0472**	0.0791	0.0423***	0.0421**	0.163***	0.0482***
	(0.0185)	(0.0575)	(0.0156)	(0.0211)	(0.0593)	(0.0173)
Constant	-21.75***	-21.19**	-27.01***	-20.11***	-38.04***	-28.17***
	(3.431)	(10.50)	(2.908)	(4.029)	(10.77)	(3.318)
Observations	134	134	134	111	111	111
R^2	0.061	0.018	0.064	0.044	0.085	0.077
P-val $\beta = 1/6$	0.000	0.130	0.000	0.000	0.957	0.000
PANEL B: Unconstrained model						
$Freq/\bar{F}$	-10.63***	-30.41***	-6.949***	-12.19***	-36.28***	-7.860***
	(1.623)	(5.072)	(0.917)	(2.170)	(5.770)	(1.061)
$Kurt/\bar{K}$	3.814***	1.335	3.585***	1.764	3.386	3.866**
	(0.906)	(3.603)	(1.427)	(1.314)	(4.366)	(1.904)
Constant	-9.623***	16.78**	-18.88***	-4.925*	13.35**	-18.72***
	(2.060)	(6.987)	(1.946)	(2.570)	(6.362)	(2.781)
R^2	0.641	0.528	0.373	0.613	0.681	0.372
Observations	134	134	134	111	111	111
P-val $\beta_f = -\beta_k$	0.001	0.000	0.016	0.000	0.000	0.047

Note: This table reports OLS results of the constrained model (equation (13)) for CPI products relating product-specific $CIR_T^{P_j}$ (calculated for the horizon T=36 months and expressed in %) to the ratio $Kurt/freq$ and OLS results of the unconstrained model (equation (14)) relating product-specific $CIR_T^{P_j}$ (expressed in %) to the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$. In Case 1, we have removed goods of three broad sectors where sales concentrate (COICOP01.1 Food, COICOP03 Clothing/Footwear, and COICOP05 Furnishing goods). In Case 2, we have removed products for which the share of sales and promotions represent more than 11% of all price changes (this threshold corresponds to the median of this ratio over all CPI products). Product-fixed effects are not included. Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 4: Regression Results - "Unconstrained" Specification - 36-month horizon

Identification	Cholesky		Cholesky		High-Freq. IV	
Long-run Restriction	Yes		No		Yes	
Product FE	No	Yes	No	Yes	No	Yes
PANEL A: PRODUCER PRICES						
$Freq/\bar{F}$	-7.366**	-3.699	-22.35*	-8.245	-6.241**	-3.610
	(3.136)	(2.432)	(11.50)	(10.15)	(2.827)	(2.575)
$Kurt/\bar{K}$	8.864**	5.788	24.19*	21.33	7.065**	3.930
	(4.257)	(3.739)	(14.53)	(13.60)	(3.300)	(3.305)
Constant	-20.45***	-15.93***	-20.79	-38.60***	-26.68***	-23.04***
	(4.488)	(3.867)	(15.44)	(13.13)	(3.234)	(6.445)
Observations	118	118	118	118	118	118
R^2	0.211	0.553	0.164	0.483	0.205	0.462
P-val $\beta_f = -\beta_k$	0.720	0.621	0.897	0.383	0.790	0.916
PANEL B: CONSUMER PRICES						
$Freq/\bar{F}$	-7.260**	-12.06***	-23.58***	-31.08***	-4.989***	-6.636***
	(2.831)	(1.537)	(7.547)	(6.082)	(1.409)	(0.944)
$Kurt/\bar{K}$	4.724***	3.003*	2.285	-3.530	3.541***	2.687**
	(1.689)	(1.567)	(3.641)	(3.461)	(1.161)	(1.076)
Constant	-14.09***	5.587*	4.675	34.30***	-21.21***	-12.69***
	(3.560)	(3.156)	(8.494)	(10.85)	(2.068)	(1.848)
Observations	223	223	223	223	223	223
R^2	0.215	0.723	0.257	0.576	0.176	0.793
P-val $\beta_f = -\beta_k$	0.469	0.000	0.0136	0.000	0.417	0.005

Note: this table reports results of OLS regressions (equation (14)) where the dependent variable is the product-specific $CIR_T^{P_j}$ (calculated for the horizon T=36 months, and expressed in %) and the right-hand-side variables are the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$. Product fixed effects are defined at the 2-digit level for both CPI and PPI products (i.e. 38 product fixed effects for the CPI, and 24 in the case of the PPI). Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 5: Regression Results - Placebo Specification - 36-month horizon

Identification	Cholesky		Cholesky		High-Freq. IV	
Long-run Restriction	Yes		No		Yes	
Product FE	No	Yes	No	Yes	No	Yes
PANEL A: PRODUCER PRICES						
Kurt/Freq	0.201**	0.110	0.513	0.222	0.188**	0.0987
	(0.0960)	(0.0782)	(0.312)	(0.275)	(0.0768)	(0.0666)
Mean	-1.430	-1.350	-10.90	-4.844	-1.262	1.273
	(1.771)	(1.694)	(6.644)	(7.412)	(1.522)	(1.677)
Skewness	-1.869	-3.824	-16.72	-6.397	-2.207	-3.327
	(4.613)	(4.452)	(17.15)	(20.29)	(2.718)	(3.625)
SD	-1.355	0.445	-6.436	-4.213	-0.182	2.197
	(2.509)	(2.335)	(8.512)	(8.616)	(2.011)	(1.728)
Constant	-21.45**	-19.54	-10.56	-11.56	-32.89***	-38.56***
	(9.551)	(12.41)	(30.11)	(44.33)	(7.834)	(11.33)
Observations	118	118	118	118	118	118
R^2	0.100	0.537	0.075	0.471	0.113	0.465
PANEL B: CONSUMER PRICES						
Kurt/Freq	-0.0199	0.0231	0.0866	0.158**	0.0116	0.0299**
	(0.0223)	(0.0218)	(0.0562)	(0.0609)	(0.0140)	(0.0146)
Mean	1.333**	1.650**	-0.882	0.546	0.132	0.759*
	(0.581)	(0.692)	(1.844)	(1.860)	(0.438)	(0.427)
Skewness	5.920**	3.762	17.76**	4.780	4.161**	3.868**
	(2.717)	(2.869)	(8.502)	(9.304)	(1.723)	(1.796)
SD	-0.823	-0.0602	3.128	5.094**	-0.528	0.251
	(0.719)	(0.871)	(1.899)	(2.190)	(0.532)	(0.517)
Constant	-8.657	-11.05	-42.46**	-60.04***	-18.77***	-23.17***
	(7.080)	(6.737)	(18.92)	(18.31)	(4.394)	(4.001)
Observations	223	223	223	223	223	223
R^2	0.037	0.455	0.049	0.367	0.027	0.658

Note: this table reports results of OLS regressions (equation (13)) where the dependent variable is the product-specific $CIR_T^{P_j}$ (calculated for the horizon T=36 months and expressed in %) and the right-hand-side variables include the product-specific ratio $Kurt/freq$ but also three other moments of the product-specific price change distribution: the average price change $Mean$, the skewness of price changes $Skewness$, and the standard deviation of price changes $StandardDev.$. Product fixed effects are defined at the 2-digit level for both CPI and PPI products (i.e. 38 product fixed effects for the CPI, and 24 in the case of the PPI). Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

ONLINE APPENDIX

Empirical Investigation of a Sufficient Statistic for Monetary Policy Shocks

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A The equilibrium representation as a MFG

We let $u(x,t)$ denote firm's value function and $m(x,t)$ denote the cross sectional distribution of $x \in [\underline{x}(t), \bar{x}(t)]$. The equilibrium for this problem can be represented as a mean field game, as in Alvarez, Lippi, and Souganidis (2022). Consider the problem of a firm that takes as given a path for $\{\mathcal{W}(t)\}$ for $t \in [0,T)$, and a terminal value function $u_T(x)$, allowing for $T \to \infty$.

Given initial and terminal conditions, $\{m_0, u_T\}$, a mean field game consists of the functions u and m, mapping $\mathbb{R} \times [0,T]$ to $\mathbb{R}$, and functions $\underline{x}, \bar{x}, x^*, X$ mapping $[0,T]$ to $\mathbb{R}$. The equilibrium is given by the solution of two partial differential equations: the Hamilton-Jacobi Bellman (HJB) for the firm's value function u, and the Kolmogorov Forward Equation (KFE)) for the cross sectional density m. For all $t \in [0,T]$ and for all $x \in [\underline{x}(t), \bar{x}(t)]$ the equations are

$$0 = u_t\left(x,t\right) - \rho u(x,t) + \frac{\sigma^2}{2} u_{xx}(x,t) + F(x, \mathcal{W}(t)) + \zeta\left[u(x^*(t),t) - u(x,t)\right] \tag{16}$$

$$0 = -m_t\left(x,t\right) + \frac{\sigma^2}{2} m_{xx}(x,t) - \zeta m(x,t) \quad \text{ and } x \neq x^*(t) \tag{17}$$

where the flow cost $F(x, \mathcal{W}(t))$ was given in equation (8) for all $t \in [0,T]$ and the path $\mathcal{W}(t)$ is given. The function $u(x,t)$ solves the HJB in equation (16), with appropriate boundary conditions, given in (19) - (21). Because of the time dependence of $\mathcal{W}(t)$ the value function u depends on time. The density $m(x,t)$ satisfies the KFE in (17), with appropriate boundary conditions given in (22) - (24). The density is needed to compute the average value of the gap

$$X(t) = \int_{\underline{x}(t)}^{\bar{x}(t)} x\, m(x,t) dx \quad \text{and} \quad x^*(t) = \arg\min_x u(x,t) \tag{18}$$

Together with $\mathcal{W}(t)$ it gives the aggregate output deviation from steady state as the inverse of the average price gap.

The boundary and terminal conditions for u are:

$$u_x\left(\bar{x}(t),t\right) = u_x\left(\underline{x}(t),t\right) = u_x\left(x^*(t),t\right) = 0 \text{ for all } t \in [0,T] \tag{19}$$

$$u\left(\bar{x}(t),t\right) = u\left(\underline{x}(t),t\right) = u\left(x^*(t),t\right) + \psi \text{ for all } t \in [0,T] \tag{20}$$

$$u(x,T) = u_T(x) \text{ for all } x \tag{21}$$

The boundary and initial conditions for m are

$$0 = m\left(\bar{x}(t), t\right) = m\left(\underline{x}(t), t\right) \text{ for all } t \in [0, T] \tag{22}$$

$$1 = \int_{\underline{x}(t)}^{\bar{x}(t)} m(x, t) dx \text{ for all } t \in [0, T] \tag{23}$$

$$m(x, 0) = m_0(x) \text{ for all } x \tag{24}$$

We solve for the equilibrium dynamics using the method described in Alvarez, Lippi, and Souganidis (2022). The setup here is simpler in that the two pde's are not coupled, since $\mathcal{W}(t)$ is exogenously given.[35]

A.1 Optimal decisions and aggregation after a shock

We consider a perturbation of the steady state indexed by the size of the shock δ, and focus on the local dynamics around $\delta = 0$. We follow the perturbation method in Alvarez, Lippi, and Souganidis (2022) (ALS henceforth) and solve for the derivatives of the objects of interest, from which we construct the full solution up to the first order.

We consider an equilibrium with $\{\bar{x}(t, \delta), \underline{x}(t, \delta),\ x^*(t, \delta),\ X(t, \delta),\ u(x, t, \delta), m(x, t, \delta)\}$, where δ indexes the perturbation of the initial condition for a given $\omega(t)$. We differentiate all the equilibrium objects with respect to δ and evaluate them at $\delta = 0$. For all $t \in [0, T]$ we denote these derivatives as follows:

$$\begin{aligned}
v(x, t) &\equiv \frac{\partial}{\partial \delta} u(x, t, \delta)|_{\delta=0} \text{ for all } x \in [-1, 1] \\
n(x, t) &\equiv \frac{\partial}{\partial \delta} m(x, t, \delta)|_{\delta=0} \text{ for all } x \in [-1, 1], x \neq 0 \\
\bar{z}(t) &\equiv \frac{\partial}{\partial \delta} \bar{x}(t, \delta)|_{\delta=0}\,,\ \underline{z}(t) \equiv \frac{\partial}{\partial \delta} \underline{x}(t, \delta)|_{\delta=0}\,,\ z^*(t) \equiv \frac{\partial}{\partial \delta} x^*(t, \delta)|_{\delta=0} \text{ and} \\
Z(t) &\equiv \frac{\partial}{\partial \delta} X(t, \delta)|_{\delta=0}
\end{aligned}$$

Once these derivatives are solved for, all objects of interest can be computed as e.g. $u(x, t, \delta) \approx \tilde{u}(x) + \delta v(x, t)$, $m(x, t, \delta) \approx \tilde{m}(x) + \delta n(x, t)$, or $X(t, \delta) \approx \delta Z(t)$ since we consider a perturbation around the steady state, where the approximation error is of order smaller than δ.

We study the evolution of the derivative of the value function, $v(x, t)$, as function of the path of the average price gap $\{Z(t)\}$. To do so we first obtain the pde and boundary conditions that $v(\cdot, t)$ satisfies. We then look for an explicit solution of $v(\cdot, t)$, which we use to compute the thresholds $\{\underline{z}(t), z^*(t), \bar{z}(t)\}$ as a function of the path of $\{\omega(t)\}$ (see section 3 in ALS for details).

We use the analytic solution in Proposition 4 of ALS to solve for the optimal path of the thresholds following the shock. The analysis shows that $\underline{z}(t) = \bar{z}(t)$ so that the width of the inaction region remains constant (the upper and lower thresholds move by the same amount). We

[35] The problem in Alvarez, Lippi, and Souganidis (2022) has a fixed point structure since $\mathcal{W}(t) = X(t)$, and thus the problem cannot be solved recursively. In words, decisions depend on aggregates, which in turn depend on decisions. Here instead decisions depend on the exogenous path $\mathcal{W}(t)$. This allows us to solve for equation (16) without knowing $m(x, t)$.

have

$$\bar{z}(t) = -\bar{A} \int_t^T \bar{H}(\tau - t)\omega(\tau)d\tau \text{ for all } t \in [0, T) \tag{25}$$

$$z^*(t) = -A^* \int_t^T H^*(\tau - t)\omega(\tau)d\tau \text{ for all } t \in [0, T) \tag{26}$$

where $\bar{H}$ and H^* are defined as:

$$\bar{H}(s) \equiv \sum_{j=1}^{\infty} e^{-(\eta^2+(j\pi)^2)\,k\,s} > 0 \ , \ H^*(s) \equiv \sum_{j=1}^{\infty} e^{-(\eta^2+(j\pi)^2)\,k\,s}(-1)^j < 0 \text{ for all } s > 0$$

and

$$\bar{A} \equiv k\, \tfrac{2\eta^2}{[1-\eta \coth(\eta)]} < 0 \quad , \quad A^* \equiv k\, \tfrac{2\eta^2}{[1-\eta \operatorname{csch}(\eta)]} > 0$$

and for notation convenience we defined the constants

$$k \equiv \tfrac{\sigma^2}{2} \quad , \quad \eta \equiv \sqrt{\frac{\rho + \zeta}{k}} \quad , \quad \ell \equiv \sqrt{\frac{\zeta}{k}}$$

Equation (25) and (26) show that the optimal policy thresholds at time t are determined by the presented discounted value of the future evolution of the aggregate shock $\omega(t)$.

Likewise, we use proposition 8 in ALS to compute the mean value of the price gap following the shock (we note that without loss of generality we normalize $\bar{x} = 1$ in ALS, this amounts to a rescaling of monetary shocks). We also define the "calvoness index" $\ell \equiv \sqrt{\frac{\zeta}{k}}$, so that $\ell \to 0$ yields the canonical menu cost problem and $\ell \to \infty$ gives the Calvo model. We have

$$Z(t) = Z_0(t) + 4k \int_0^t G^*(t - \tau)z^*(\tau)d\tau + 4k \int_0^t \bar{G}(t - \tau)\bar{z}(\tau)d\tau \tag{27}$$

for all $t \in [0, T]$ and where $\bar{G}$, G^* and Z_0, are defined as

$$\bar{G}(s) \equiv -\tilde{m}_x(1) \sum_{j=1}^{\infty} e^{-(\ell^2+(j\pi)^2)\,k\,s} > 0 \quad \text{and} \quad G^*(s) \equiv -\tilde{m}_x(0^+) \sum_{j=1}^{\infty} (-1)^{j+1} e^{-(\ell^2+(j\pi)^2)k\,s} > 0$$

for all $s \geq 0$, $\tilde{m}_x(1)$ and $\tilde{m}_x(0^+)$ are constants given in proposition 5 and

$$Z_0(t) \equiv 2 \sum_{j=1}^{\infty} \frac{\ell^2}{\ell^2 + (j\pi)^2} \left(\frac{(-1)^j \left(1 + e^{2\ell}\right) - 2e^{\ell}}{\left(1 - e^{\ell}\right)^2} \right) e^{-(\ell^2+(j\pi)^2)kt} \quad \text{for} \quad \ell > 0 \ . \tag{28}$$

The component $Z_0(t)$ is the solution that originates from a once and for all shock to the money supply. For instance a permanent increase of the money stock. This shock is analyzed in details in Alvarez and Lippi (2022). The other two components of the aggregate response in equation (27) capture the effect of the transitory shock $\mathcal{W}$, affecting aggregate effect through its effect on the firm's optimal policy z^* and $\bar{z}$. Any combination of the two shocks can be analyzed. For instance we can assume $Z_0(t) = 0$ for all t, in which case we will focus on the effects of a transitory shock without a long run effect. Finally we note that varying the value of ℓ allows us to consider different economies, from Golosov Lucas to the Calvo model.

A.2 Aggregate dynamics following the shock

The equilibrium path for $Z(t)$ is readily obtained by replacing equation (25) and equation (26) into equation (27). A straightforward application of proposition 7 in ALS gives

$$Z^{\nu,\omega}(t) = Z_0^{\nu}(t) - \int_0^T K(t,s)\omega(s)ds \text{ all } t \in [0,T] \tag{29}$$

where Z_0^{ν} is given by equation (28) and where the kernel K is:

$$K(t,s) = \\ 4\sum_{j=1}^{\infty}\sum_{i=1}^{\infty}\left[\bar{A}_\ell - A_\ell^*\,(-1)^{j+i}\right]\frac{\left[e^{[(j\pi)^2+(i\pi)^2+\eta^2+\ell^2]k(t\wedge s)}-1\right]e^{-(j\pi)^2kt-\ell^2kt-(i\pi)^2ks-\eta^2ks}}{(j\pi)^2+(i\pi)^2+\eta^2+\ell^2} \tag{30}$$

with $\bar{A}_\ell \equiv -\tilde{m}_x(1)\,\bar{A} < 0$ and $A_\ell^* \equiv -\tilde{m}_x(0^+)\,A^* > 0$, $\bar{A} < 0$ and $A^* > 0$ are constants introduced above and $\tilde{m}_x(1) = -\frac{\ell^2 e^\ell}{(1-e^\ell)^2}$, $\tilde{m}_x(0^+) = -\frac{\ell^2\,(1+e^{2\ell}}{2(1-e^\ell)^2}$, see Alvarez, Lippi, and Souganidis (2022).

From equation (2) the equilibrium path for output is then given by $Y(t) \equiv \delta y(t) + o(\delta)$ where

$$y^{\nu,\omega}(t) = \omega(t) - Z^{\nu,\omega}(t) \tag{31}$$

We summarize our main result as follows:

PROPOSITION 2. Using equation (29) and equation (31) the output dynamics following a permanent shock of size δ and a transitory shock given by the sequence $W(t) = \delta\omega(t)$ is given by $Y(t) = \delta y(t) + o(\delta)$ where

$$y(t) = y_0(t) + \omega(t) + \int_0^T K(t,s)\omega(s)ds \tag{32}$$

This proposition is quite useful. It gives the output dynamics implied by a monetary shock made of two components: a permanent one (δ), and a transitory one $\omega(t)$. The response to the permanent component $y_0(t) = -Z_0(t)$ was solved for in Alvarez and Lippi (2022) and given in equation (28).

The response to the transitory component, given by the second and third terms on the right hand side of equation (32), is new. Next we use this result to discuss a few cases of interest.

A.3 The response to shocks with a transitory component

In this section we use Proposition 2 to analyze various cases of interest, with the final aim to understand how the introduction of a transitory component affects the response to the shocks. First we present an analytic solution for the case where transitory shock $\omega(t)$ follows an exponential path. Second we analytically characterize the cumulative impulse response function (CIR) to analyze how the presence of the transitory component affects the sufficient statistics result discussed in Alvarez, Le Bihan, and Lippi (2016) and Baley and Blanco (2021). The sufficient statistic results of these papers are derived under the assumption that the shock is permanent. We inspect the robustness of the proposition as a function of the duration of the transitory shocks. The proposition continues to hold (i.e. kurtosis over frequency is a sufficient statistic for the Calvo-plus models) when the half-life of the shock is small as well when it diverges (i.e. the shock is permanent). For the intermediate cases we provide a bound on the accuracy of the proposition.

A transitory component with exponential decay. We consider a transitory shock parametrized as $\omega(t) = \omega_o e^{-\gamma t}$. An immediate application of Proposition 2 gives

$$y(t) = y_o(t) + w_o \left(e^{-\gamma t} + \int_0^T K(t,s)\; e^{-\gamma s}\; ds \right) \tag{33}$$

where $y_o(t) \equiv -Z_0(t)$, the response to the permanent shock which was given in equation (28).

Since $K(t,s)$ also involves terms that have an exponential decay in time, we can solve this impulse response in closed form. To slightly simplify the algebra we focus on the case with zero discounting $\rho \to 0$ (so that $\ell^2 = \eta^2 = \zeta/k$) and an infinite horizon $T \to \infty$ (see the proof for details and more general cases). We have the following result

Proposition 3. Consider $T \to \infty$ and $\rho \to 0$ and a transitory shock $\omega(t) = \omega_0 e^{-\gamma t}$. We have:

$$y(t) = y_o(t) + \; w_o\, e^{-\gamma t} \left(1 + 4 \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} \frac{k\left[\bar{A}_\ell - A^*_\ell\, (-1)^{j+i}\right] \left(1 - e^{-[(j\pi)^2 + \ell^2]kt + \gamma t}\right)}{\left((j\pi)^2 k + \ell^2 k - \gamma\right)\left((i\pi)^2 k + \ell^2 k + \gamma\right)} \right) \tag{34}$$

where $\bar{A}_\ell$ and A^*_ℓ are constants defined above.

It is convenient to write the infinite sums using Euler's formula as follows

$$\begin{aligned} y(t) &= y_o(t) + \; w_o\, e^{-\gamma t} \\ &\quad + w_o\, 4 \bar{A}_\ell \left[\frac{\sqrt{\ell^2 + \gamma/k}\, \coth(\sqrt{\ell^2 + \gamma/k}) - 1}{2(\ell^2 + \gamma/k)} \right] \sum_{j=1}^{\infty} \frac{\left(e^{-\gamma t} - e^{-[(j\pi)^2 + \ell^2]kt}\right)}{\left((j\pi)^2 k + \ell^2 k - \gamma\right)} \\ &\quad - w_o\, 4 A^*_\ell \left[\frac{\sqrt{\ell^2 + \gamma/k}\, \operatorname{csch}(\sqrt{\ell^2 + \gamma/k}) - 1}{2(\ell^2 + \gamma/k)} \right] \sum_{j=1}^{\infty} \frac{\left(e^{-\gamma t} - e^{-[(j\pi)^2 + \ell^2]kt}\right)(-1)^j}{\left((j\pi)^2 k + \ell^2 k - \gamma\right)} \end{aligned} \tag{35}$$

The proposition is useful. It allows us to compute the impulse response in an efficient way and to do comparative statics.

A.4 Proofs

Proof. (of Proposition 1) Now we compute the CIR. For that we define:

$$\begin{aligned} CIR &= \int_0^\infty y_o(t) dt + \int_0^\infty w_o \left(e^{-\gamma t} + \int_0^T K(t,s)\; e^{-\gamma s}\; ds \right) dt \\ &= CIR_0 + w_o \left(\frac{1}{\gamma} + \int_0^\infty Q(t) dt \right) \end{aligned}$$

We now compute $\int_0^\infty Q(t)dt$.

$$\int_0^\infty Q(t)dt = 4\sum_{j=1}^\infty\sum_{i=1}^\infty \frac{\left[\bar{A}_\ell - A_\ell^*\left(-1\right)^{j+i}\right]}{(j\pi)^2+(i\pi)^2+\eta^2+\ell^2}\ \int_0^\infty k_{i,j}^\infty(t)\ dt$$

where

$$k_{i,j}^\infty(t) = \left[e^{-\gamma t} - e^{-[(j\pi)^2+\ell^2]kt}\right]\left[(j\pi)^2+(i\pi)^2+\eta^2+\ell^2\right]\left[\frac{k}{\left[(j\pi)^2k+\ell^2k-\gamma\right]\left[(i\pi)^2k+\eta^2k+\gamma\right]}\right]$$

Thus, after tedious algebra

$$\int_0^\infty Q(t)dt = 4\sum_{j=1}^\infty\sum_{i=1}^\infty \left[\bar{A}_\ell - A_\ell^*\left(-1\right)^{j+i}\right]\frac{k}{\gamma}\frac{1}{\left(\left[(i\pi)^2+\eta^2\right]k+\gamma\right)}\frac{1}{\left(\left[(j\pi)^2+\ell^2\right]k\right)}$$

Proof. (of Proposition 3). The transitory component of output is the sum of two parts: $w_o e^{-\gamma t} + w_o Q(t)$ where

$$Q(t) = 4\sum_{j=1}^\infty\sum_{i=1}^\infty \left[\bar{A}_\ell - A_\ell^*\left(-1\right)^{j+i}\right]\int_0^T \frac{\left[e^{[(j\pi)^2+(i\pi)^2+\eta^2+\ell^2]k(t\wedge s)}-1\right]e^{-(j\pi)^2kt-\ell^2kt-(i\pi)^2ks-\eta^2ks-\gamma s}}{(j\pi)^2+(i\pi)^2+\eta^2+\ell^2}ds$$

Next we solve for $Q(t)$ analytically. Define

$$k_{i,j}(t) \equiv \int_0^T \left[e^{[(j\pi)^2+(i\pi)^2+\eta^2+\ell^2]k(t\wedge s)}-1\right]e^{-(j\pi)^2kt-\ell^2kt-(i\pi)^2ks-\eta^2ks-\gamma s}ds$$

so that

$$Q(t) = 4\sum_{j=1}^\infty\sum_{i=1}^\infty \frac{\left[\bar{A}_\ell - A_\ell^*\left(-1\right)^{j+i}\right]}{(j\pi)^2+(i\pi)^2+\eta^2+\ell^2}\ k_{i,j}(t)$$

Note that

$$\begin{aligned} k_{i,j}(t) \equiv &\int_0^t \left[e^{[(j\pi)^2+(i\pi)^2+\eta^2+\ell^2]ks}-1\right]e^{-(j\pi)^2kt-\ell^2kt-(i\pi)^2ks-\eta^2ks-\gamma s}ds \\ &+\int_t^T \left[e^{[(j\pi)^2+(i\pi)^2+\eta^2+\ell^2]kt}-1\right]e^{-(j\pi)^2kt-\ell^2kt-(i\pi)^2ks-\eta^2ks-\gamma s}ds \end{aligned}$$

After tedious algebra (integrating and collecting terms) we can write

$$\begin{aligned} k_{i,j}(t) = &\ \frac{e^{-\gamma t}-e^{-[(j\pi)^2+\ell^2]kt}}{[(j\pi)^2+\ell^2]k-\gamma} \\ &-\frac{e^{-[(i\pi)^2k+\eta^2k+\gamma]T+[(i\pi)^2+\eta^2]kt}-e^{-\gamma t}}{(i\pi)^2k+\eta^2k+\gamma} \\ &+\frac{e^{-[(i\pi)^2k+\eta^2k+\gamma]T-[(j\pi)^2+\ell^2]kt}-e^{-[(j\pi)^2+\ell^2]kt}}{(i\pi)^2k+\eta^2k+\gamma} \end{aligned}$$

When $T \to \infty$ we have

$$k^{\infty}_{i,j}(t) = \frac{e^{-\gamma t} - e^{-[(j\pi)^2+\ell^2]kt}}{(j\pi)^2 k + \ell^2 k - \gamma} + \frac{e^{-\gamma t} - e^{-[(j\pi)^2+\ell^2]kt}}{(i\pi)^2 k + \eta^2 k + \gamma} \quad .$$

B FAVAR estimation

The Factor Augmented Vector Autoregression (FAVAR) was originally developed by Bernanke, Boivin, and Eliasz (2005) and by Boivin, Giannoni, and Mihov (2009). Stock and Watson (2016) provide also a clear explanation of the model.

Let i_t be a vector of observable economic variables with dimension M x 1, $M \geq 1$, and let $\tilde{F}_t$ be a vector of unobserved factors with dimension K x 1, $K \geq 1$.[36] Assume that the dynamics of the economy is driven by $(Y_t', \tilde{F}_t')$ which follows the transition equation:

$$\begin{bmatrix} \tilde{F}_t \\ i_t \end{bmatrix} = \Phi(L) \begin{bmatrix} \tilde{F}_{t-1} \\ i_{t-1} \end{bmatrix} + v_t \tag{36}$$

where $\Phi(L)$ is a lag polynomial of finite order and v_t is an error term with zero mean and covariance matrix Q. While equation (36) has a VAR form, given that F_t is unobserved we cannot directly estimate equation (36). However, the factors $\tilde{F}_t$ are interpreted as representing forces that potentially affect many economic variables from which we can estimate the factors. Indeed, assume that a large number of time series X_t, called informational time series, are related to the observed variables i_t and to the unobservable factors $\tilde{F}_t$ by the following equation:

$$X_t = \Lambda F_t + e_t \tag{37}$$

where $F_t \equiv [\tilde{F}_t \; i_t]'$ and e_t is a vector N x 1 of error terms with zero mean.[37] Notice that the number of informational time series, N, must be large which means N is much larger than the number of variables that drives the economy (F_t and i_t), i.e. $N > K + M$, and potentially N can be larger than the number of observations in the time dimension, T. Moreover, notice that F_t can always capture arbitrary lags of fundamental factors, thus it is not restrictive to assume that X_t depends only on the current values of the factors.[38]
Under the above assumptions, it is possible to estimate the model, using a two-step approach.[39] In the first step, the common factors are estimated extracting the first K principal components, $\hat{C}^{(0)}$, from the information variables, X_t. Indeed, as shown by Stock and Watson (2002), for N large enough and if the number of principal components used is as least as large as the true number of factors, the principal components of X_t span the space generated by the factors $\tilde{F}$ and the observable variables i_t; thus, the principal components represent independent but arbitrary linear combinations of $\tilde{F}_t$ and i_t. However, we want that these combinations do not depend on i_t and that they are only independent combinations of the factors. For this reason, the factors are estimated as follows. Regress X_t on $\hat{C}^{(0)}$ and i_t to obtain $\hat{B}_r^{(0)}$, the coefficient of i_t. Then compute $\tilde{X}_t^{(0)} = X_t - \hat{B}_r^{(0)} i_t$ and estimate $\hat{C}^{(1)}$ as the first K principal components of $\tilde{X}_t^{(0)}$. Iterate

[36] We adopt the notation 'i_t' as in our application the observable factor reduces to the interest rate.

[37] If factors are estimated using a principal components analysis, errors can display a small amount of cross-correlation that must vanish as N goes to infinity. See Stock and Watson (2002) for a detailed discussion.

[38] For this reason Stock and Watson (1999) refer to equation (37) as a dynamic factor model.

[39] An alternative to estimate the model is to use a single-step Bayesian likelihood approach.

until convergence of $\hat{B}_r^{(i)}$ to obtain the desired estimated factors, $\hat{\hat{F}}_t$. The second step consists in estimating equation (36) as a structural VAR[40], replacing F_t with their estimated counterpart $\hat{F}_t \equiv [\hat{\hat{F}}_t \; i_t]'$. Indeed, we can rewrite equation (36) as

$$\hat{F}_t = \Phi(L)\hat{F}t - 1 + v_t \tag{38}$$

where $\hat{F}_t^{+} \equiv [\hat{F}_t \; i_t]'$. Assuming $v_t = H\epsilon_t$, it is clear that equation (38) can be treated as a structural VAR.

The final step we are interested in is to estimate the IRFs of X_t. Consider again equation (38) and assume that the MA representation exists. Denoting the MA coefficient with $\Psi(L)$, we obtain

$$\hat{F}_t = \Psi(L)H\epsilon_t \tag{39}$$

Moreover, using $\hat{F}_t$ instead of F_t in equation (37) and replacing in this equation equation (39), we get

$$X_t = \Lambda\Psi(L)^{-1}H\epsilon_t + e_t \tag{40}$$

Equation (40) links the information variables, X_t, to the shocks and provides the theoretical framework to retrieve the IRFs of X_t. However, in practice, the IRFs of X_t are not estimated using the MA representation and, thus, equation (40). Indeed, let $\widehat{IRF(A)}$ be the estimated IRFs of the time series A_t to a given shock. The IRFs of X_t is calculated as

$$\widehat{IRF(X)} = \widehat{\beta} * \widehat{IRF(\hat{F})} \tag{41}$$

where $\widehat{IRF(\hat{F})}$ is the VAR estimated IRF of $\hat{F}_t$ and $\widehat{\beta}$ is the estimated coefficient of the regression of X_t on $\hat{F}_t$.

Details on variables in the FAVAR We include three types of "informative time series" in vector X_t: (i) macroeconomic data for France including aggregate industrial production, aggregate producer price index (PPI), the aggregate harmonized index of consumer prices (HICP), unemployment rate, (ii) financial and monetary variables relevant for the euro area including the monetary aggregate M3 in the euro area, the value of banknotes in circulation in the euro area, the euro exchange rate with respect to US dollar, yen, UK pound sterling, Swiss franc, Chinese Yuan Renminbi (iii) highly disaggregated French series of industrial production indices (IPI), producer price indices (PPI) and consumer price indices (CPI), as well as some available disaggregated series for monthly consumption (16 broad categories of consumptions at an intermediate aggregation level, including, for instance, durables consumption, manufacturing goods consumption). As regards product-specific monthly price series, CPI price indices are available at the 5-digit level of the ECOICOP classification (e.g. '01.1.1.1' 'Rice') whereas PPI price indices in the manufacturing sector are available at the 4-digit level of the NACE rev2 classification of sectors (e.g. '08.11' 'Quarrying of ornamental and building stone, limestone, gypsum, chalk and slate'). Overall, we use 223 product-specific consumer price indices covering both goods and services and 118 producer price indices covering the manufacturing sector.

[40] In our application, we estimate the structural VAR using a Cholesky decomposition. However, any other approach can be used.

C A Filter for the Euribor

Our empirical goal is to estimate a monetary shock characterized by a transient impact on inflation and output. We filter the 3-month Euribor so as to ensure this property, as with unfiltered data it is not fulfilled. One possible reason why it is not fulfilled (unlike in typical VAR) is because in our sample period, on the euro area, this variable is not stationary, as depicted in Figure A.1. We exploit two criteria to choose the value of the one-sided HP filter, λ^{HP}. Both criteria are based on the behavior of the IRFs of PPI and CPI time series as λ^{HP} varies. We estimated the FAVAR model, for alternative values of filtered Euribor rates letting the one-sided HP filter parameter, λ^{HP}, vary from 6 to $4 * 10^6$. Furthermore, we retrieve the sectorial IRF using the Cholesky decomposition or the high frequency identification, both without imposing a long run restriction.

One first criterion considers the number of negative IRFs of PPI or CPI after three years, since our strong prior is that after a contractionary monetary shock, prices should decline as compared to the no-shock baseline. Thus, we are interested in estimating a FAVAR that is in line with this prediction. The top panel of Figure A.2 shows our finding under the Cholesky decomposition assumption: the blue (orange) line reports the percentage of PPI (CPI) sector with a negative IRF after three years. Both for PPI and CPI, the share of negative IRFs is maximized around the value of λ^{HP} of 500k. For λ^{HP} =500k we have around 60% of the sectors with negative IRF. Moreover, this curve is very flat for values grater than 500k and, indeed, all the results in paper are robust when using an higher value of the filter.

As a second criterion to guide our choice of λ^{HP}, we consider the arithmetic average price response of PPI or CPI products to the contractionary monetary shock after three years as a function of λ^{HP}. We expect that these two aggregate IRFs have negative values. The blue line in the botton panel of Figure A.2 shows that the average sectorial response of PPI is negative only for values λ^{HP} above 55k. Futhermore, for values larger than 55k also the average sectoral response of CPI is negative (orange line in the bottom panel of Figure A.2).

Figure A.3 reports the same statistics as Figure A.2. In this case, however, the monetary shock is identified using the instrumental variable approach. In the top panel, the blue (resp. orange) line reports the share of PPI (resp. CPI) products with a negative IRF after three years. Both lines have two maximum. The first maximum is for the value of 85k of the filter. The other one is for a value of 6. We select the value of 85k since for this value, the number of sectors with a negative IRF after three years is quite unaffected by small changes of the one-sided HP filter (which is less true for a value of 6).[41] Finally, the bottom panel of Figure A.3 reports the arithmetic average response of PPI or CPI sectors to the contractionary monetary shock after three years as a function of λ^{HP}. The blue (orange) line shows the average sectorial response of PPI (CPI). Both lines are negative for values of the one-sided HP filter close to 85k. For these reason, we choose 85k as our optimal value for λ^{HP}.

[41] We have also estimated our FAVAR and regressions using a value of 6 for the one-sided HP filter. The results are very close to the value of 85k.

Figure A.1: 3-month Euribor: period 2005-2019

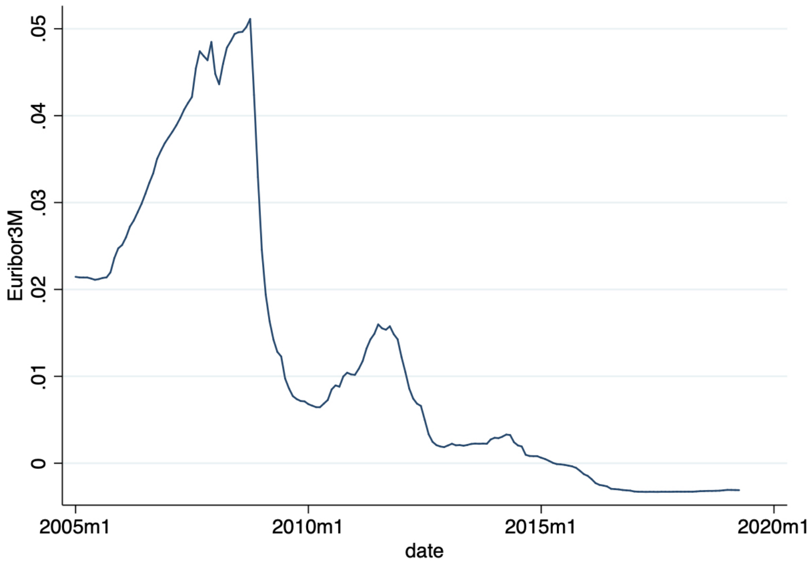
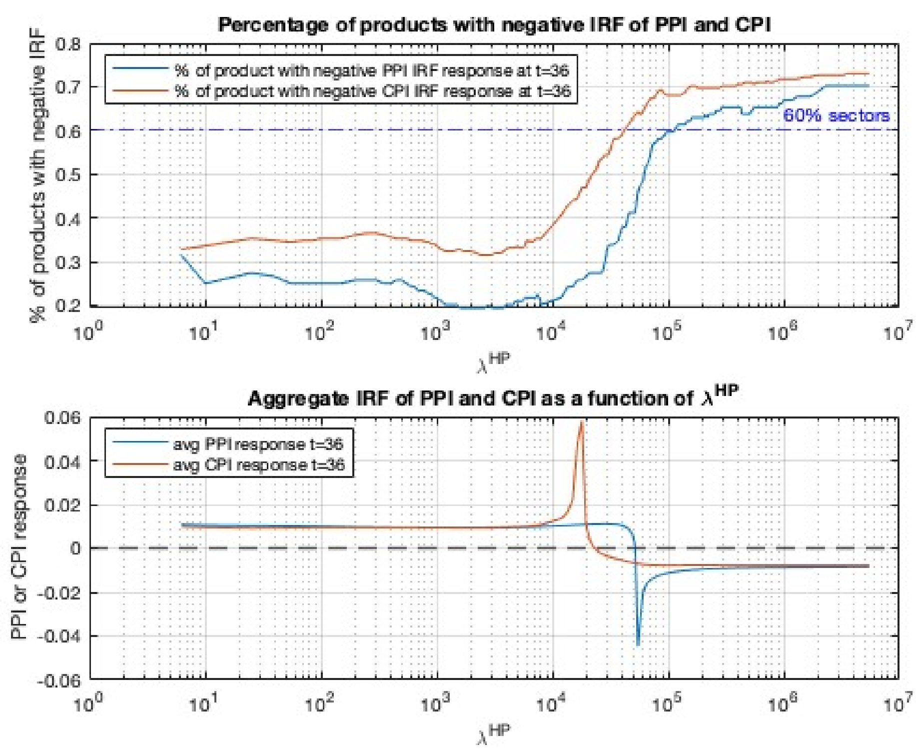
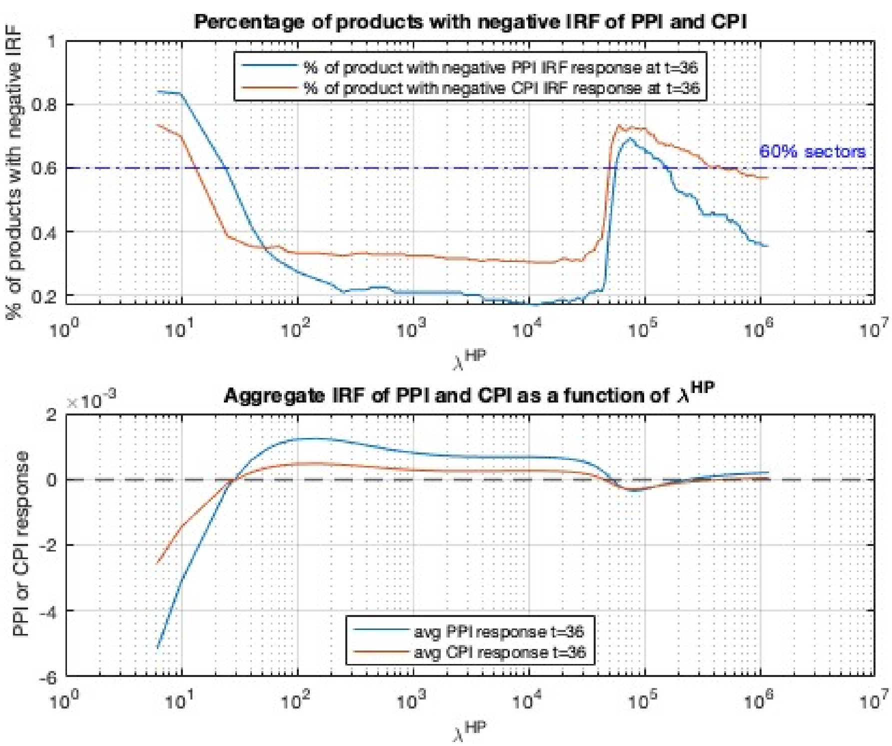


Figure A.2: Response of sectoral PPI and CPI as a function of λ^{HP} - Shock identified with Cholesky



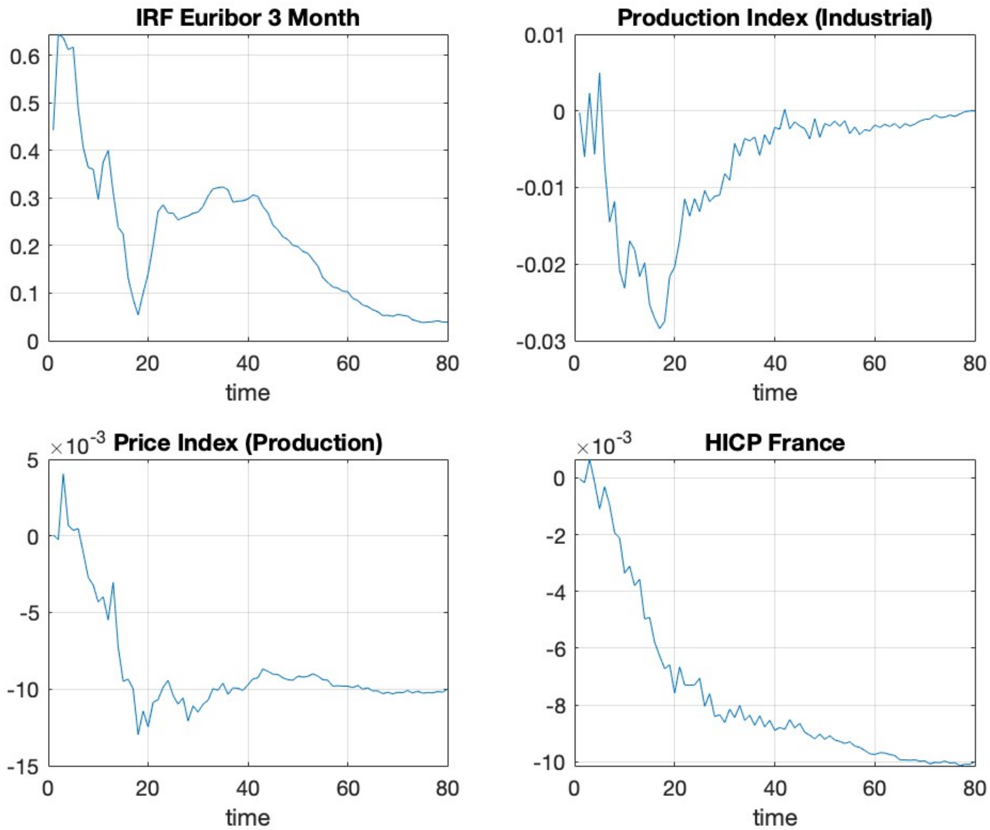
Top panel: the blue (orange) line reports the percentage of PPI (CPI) sectors with a negative IRF after three years to a contractionary monetary shock as a function of the one-sided HP filter parameter, λ^{HP}. Bottom panel: the blue (orange) line reports sectoral IRF of production (consumer) prices to a contractionary monetary shock as a function of the one-sided HP filter parameter. The monetary shock is identified using the Cholesky decomposition.

Figure A.3: Response of sectoral PPI and CPI as a function of λ^{HP} - Shock identified with IV



Top panel: the blue (orange) line reports the percentage of PPI (CPI) sectors with a negative IRF after three years to a contractionary monetary shock as a function of the one-sided HP filter parameter, λ^{HP}. Bottom panel: the blue (orange) line reports sectoral IRF of production (consumer) prices to a contractionary monetary shock as a function of the one-sided HP filter parameter. The monetary shock is identified using the IV approach.

Figure A.4: Aggregate response to a contractionary monetary policy shock (Cholesky - Long-run restriction)



y-axis: log points in deviation from the ”steady state”.
Top panel: 3-month Euribor impulse response function (IRF). Top right panel: production index IRF. Bottom left panel: production price index IRF. Bottom right panel: IRF of the harmonized index of consumer prices

Table A.1: Product-specific CIR^P : Descriptive Statistics

	Moments of the CIR distribution								
	Mean	Std. Dev.	min	5%	25%	50%	75%	95%	max
PANEL A: PRODUCER PRICES									
Cholesky - Long-run restriction									
24 months	-9.17	15.43	-95.73	-50.66	-8.71	-5.95	-3.11	0.86	17.04
36 months	-18.95	19.01	-140.67	-55.00	-18.37	-14.72	-11.45	-4.32	6.59
48 months	-29.07	20.08	-163.08	-58.73	-28.53	-24.71	-21.05	-12.93	-2.98
Cholesky - No long-run restriction									
24 months	-9.17	36.01	-238.29	-53.69	-9.36	-2.08	3.92	15.43	32.32
36 months	-18.95	64.43	-436.04	-101.12	-27.54	-5.61	5.97	30.38	66.78
48 months	-29.07	98.22	-658.55	-173.34	-41.50	-8.22	6.89	50.37	111.84
High Frequency Instrument - Long-run restriction									
24 months	-12.31	8.49	-56.38	-27.16	-12.93	-11.00	-8.60	-3.67	6.07
36 months	-25.86	16.20	-114.59	-54.08	-27.23	-22.26	-18.09	-9.64	7.40
48 months	-37.87	18.63	-133.72	-70.84	-41.06	-34.05	-28.96	-17.39	13.03
PANEL B: CONSUMER PRICES									
Cholesky - Long-run restriction									
24 months	-7.17	12.44	-90.86	-29.56	-9.05	-4.64	-1.59	4.99	14.28
36 months	-16.62	16.95	-139.36	-42.85	-16.88	-12.18	-9.21	-5.05	0.91
48 months	-27.15	19.27	-163.72	-55.56	-27.29	-21.59	-19.03	-14.14	-5.79
Cholesky - No long-run restriction									
24 months	-7.17	24.58	-201.17	-42.07	-11.77	-3.75	1.96	18.09	46.28
36 months	-16.62	46.08	-367.90	-85.59	-28.63	-12.27	3.22	35.76	105.13
48 months	-27.15	71.43	-547.10	-125.41	-47.64	-23.07	5.16	57.02	184.05
High Frequency Instrument - Long-run restriction									
24 months	-11.60	8.64	-101.25	-25.89	-11.98	-9.81	-8.36	-5.54	3.18
36 months	-22.65	13.04	-142.21	-47.50	-23.09	-19.93	-17.73	-13.57	-2.56
48 months	-34.41	14.79	-164.47	-63.69	-35.71	-31.19	-28.28	-22.65	-11.09

Note: this table reports descriptive statistics on the distribution of the product-specific CIR of prices obtanied from the the different FAVAR specifications and at 24-, 36- and 48-month horizons (expressed in %). These statistics are computed over 118 products for PPI and 223 products for CPI.

D OLS regressions - Additional Results and Robustness

Table A.2: Testing Model's Predictions

Identification	Cholesky		Cholesky		High-Freq. IV	
Long-run Restriction	Yes		No		Yes	
Product FE	No	Yes	No	Yes	No	Yes
PANEL A: PRODUCER PRICES						
Constrained model						
P-val $\beta = 1/6$	0.598	0.358	0.129	0.713	0.736	0.568
P-val $\alpha = -T$	0.080	0.000	0.671	0.795	0.617	0.215
Unconstrained model						
P-val $\beta_f = -\beta_k$	0.720	0.621	0.897	0.383	0.790	0.916
P-val $\beta_f = -\frac{\bar{K}}{6\bar{F}}$	0.355	0.758	0.122	0.710	0.528	0.745
P-val $\beta_k = \frac{\bar{K}}{6\bar{F}}$	0.302	0.722	0.177	0.218	0.430	0.875
PANEL B: CONSUMER PRICES						
Constrained model						
P-val $\beta = 1/6$	0.000	0.000	0.001	0.092	0.000	0.000
P-val $\alpha = -T$	0.000	0.000	0.004	0.000	0.000	0.000
Unconstrained model						
P-val $\beta_f = -\beta_k$	0.469	0.000	0.0136	0.000	0.417	0.005
P-val $\beta_f = -\frac{\bar{K}}{6\bar{F}}$	0.807	0.008	0.040	0.002	0.037	0.164
P-val $\beta_k = \frac{\bar{K}}{6\bar{F}}$	0.057	0.002	0.121	0.001	0.000	0.000

Note: we report p-values of Wald tests performed on the parameters of our baseline OLS regressions presented in Table 2 and Table 2. These tests correspond to model's predictions presented in equation (13) and equation (14). We perform three different tests: (i) in the constrained version of the model we test whether β (parameter associated with the ratio $Kurt/Freq$ is equal to $-\delta/6$ (where δ is the MP shock here normalised to 1%); and (ii) in the unconstrained model, we test whether the parameter associated with frequency (β_f) is equal to minus the parameter associated with kurtosis ($-\beta_k$); (iii) in the unconstrained version, we also perform tests on the parameter associated with frequency and kurtosis, they are predicted to be equal to $\frac{\bar{K}}{6\bar{F}}$ where $\bar{K}$ and $\bar{F}$ are sample averages of kurtosis and frequency.

Table A.3: Regression Results - Placebo Unconstrained Specification

Identification	Cholesky		Cholesky		High-Freq. IV	
Long-run Restriction	Yes		No		Yes	
Product FE	No	Yes	No	Yes	No	Yes
PANEL A: PRODUCER PRICES						
$Freq/\bar{F}$	-7.498**	-3.582	-23.41*	-9.097	-6.106*	-2.884
	(3.649)	(2.704)	(13.21)	(11.35)	(3.279)	(2.849)
$Kurt/\bar{K}$	6.563	6.835	12.28	23.25	6.534*	5.083
	(4.412)	(6.460)	(14.37)	(24.11)	(3.504)	(6.340)
Mean	-1.201	-0.947	-11.27*	-4.153	-0.859	1.682
	(1.578)	(1.792)	(6.648)	(8.094)	(1.359)	(1.695)
Skewness	-1.352	-2.953	-13.16	-0.718	-2.321	-3.039
	(4.615)	(5.243)	(17.37)	(22.96)	(2.548)	(4.225)
SD	-1.571	1.116	-8.880	-0.846	-0.140	2.599
	(3.092)	(3.222)	(11.23)	(12.24)	(2.501)	(2.692)
Constant	-10.92	-22.10	34.34	-31.53	-25.66*	-39.45**
	(17.21)	(19.41)	(61.81)	(72.23)	(14.43)	(18.50)
Observations	118	118	118	118	118	118
R^2	0.214	0.556	0.182	0.485	0.207	0.480
PANEL B: CONSUMER PRICES						
$Freq/\bar{F}$	-8.213***	-12.28***	-25.75***	-31.41***	-5.838***	-6.737***
	(2.957)	(1.633)	(7.247)	(5.855)	(1.520)	(1.010)
$Kurt/\bar{K}$	1.136	0.555	13.11	9.327	1.554	2.852
	(2.487)	(2.407)	(8.805)	(8.260)	(1.684)	(1.938)
Mean	-0.302	0.637	-3.952**	-1.238	-0.615*	0.269
	(0.489)	(0.552)	(1.724)	(1.690)	(0.353)	(0.339)
Skewness	6.062**	5.900**	12.70	4.792	3.738**	3.362*
	(2.511)	(2.836)	(8.709)	(9.391)	(1.526)	(1.864)
SD	-1.137	-0.296	2.980	4.881*	-0.759	0.389
	(0.714)	(0.776)	(2.144)	(2.582)	(0.662)	(0.567)
Constant	1.032	11.09	-18.45	-11.95	-10.89	-15.16**
	(8.281)	(8.526)	(26.16)	(30.05)	(7.000)	(6.076)
Observations	223	223	223	223	223	223
R^2	0.240	0.730	0.336	0.601	0.206	0.800

Note: this table reports results of OLS regressions (equation (14)) where the dependent variable is the product-specific $CIR_T^{P_j}$ (calculated for the horizon T=36 months and expressed in %) and the right-hand-side variables include the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$, but also three other moments of the product-specific price change distribution: the average price change $Mean$, the skewness of price changes $Skewness$, and the standard deviation of price changes $StandardDev.$. Product fixed effects are defined at the 2-digit level for both CPI and PPI products (i.e. 38 product fixed effects for the CPI, and 24 in the case of the PPI). Robust standard errors are reported in parentheses. Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table A.4: Regression Results: Role of sales - Consumer Prices - Placebo

	Case 1: Excluding food, clothing/footwear, furnishings			*Case 2: Products with % of sales prices below the median*		
Identification	Cholesky	Cholesky	High-Freq. IV	Cholesky	Cholesky	High-Freq. IV
Long-run Restriction	Yes	No	Yes	Yes	No	Yes
Kurt/Freq	0.0504**	0.189**	0.0513***	0.0456**	0.174***	0.0473***
	(0.0244)	(0.0755)	(0.0181)	(0.0226)	(0.0656)	(0.0157)
Mean	0.480	-3.646*	-0.472	0.161	2.199	-0.164
	(0.392)	(1.987)	(0.454)	(0.401)	(1.786)	(0.402)
Skewness	3.791	12.03	2.415	3.485	11.61	2.056
	(2.554)	(9.000)	(2.001)	(3.024)	(9.771)	(2.549)
SD	0.686	4.725*	-0.0624	0.657	4.943*	-0.592
	(0.845)	(2.502)	(0.675)	(0.983)	(2.670)	(0.963)
Constant	-26.75***	-58.79**	-26.16***	-24.14**	-72.92***	-23.20***
	(9.230)	(27.31)	(5.673)	(9.505)	(26.55)	(6.274)
Observations	134	134	134	111	111	111
R^2	0.076	0.118	0.079	0.054	0.134	0.090

Note: This table reports OLS results of the constrained model (equation (13)) for CPI products relating product-specific $CIR_T^{P_j}$ (calculated for the horizon T=36 months and expressed in %) to the ratio $Kurt/freq$ and OLS results of the unconstrained model (equation (14)) relating product-specific $CIR_T^{P_j}$ (expressed in %) to the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$. In Case 1, we have removed goods of three broad sectors where sales concentrate (COICOP01.1 Food, COICOP03 Clothing/Footwear, and COICOP05 Furnishing goods). In Case 2, we have removed products for which the share of sales and promotions represent more than 11% of all price changes (this threshold corresponds to the median of this ratio over all CPI products). Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table A.5: Baseline OLS Regression Results : Kurtosis alone - Frequency alone

Identification	Cholesky		Cholesky		High-Freq. IV	
Long-run Restriction	Yes		No		Yes	
Product FE	No	Yes	No	Yes	No	Yes
PANEL A: Producer Prices - Kurtosis alone						
$Kurt/\bar{K}$	9.148**	5.240	25.05	20.11	7.305*	3.396
	(4.435)	(3.717)	(15.44)	(13.07)	(3.761)	(3.374)
Constant	-28.10***	-17.99***	-44.00**	-43.18***	-33.17***	-25.05***
	(5.836)	(3.412)	(20.46)	(12.98)	(4.780)	(5.934)
R^2	0.031	0.521	0.020	0.469	0.027	0.421
PANEL B: Producer Prices - Freq. alone						
$Freq/\bar{F}$	-7.404**	-3.613	-22.45*	-7.926	-6.272**	-3.551
	(3.242)	(2.463)	(11.83)	(10.28)	(2.873)	(2.537)
Constant	-11.55***	-10.79***	3.500	-19.65***	-19.59***	-19.55***
	(2.463)	(2.717)	(9.020)	(7.224)	(2.181)	(6.535)
R^2	0.182	0.545	0.146	0.474	0.180	0.457
Observations	118	118	118	118	118	118
PANEL C: Consumer Prices - Kurtosis alone						
$Kurt/\bar{K}$	5.567***	5.097**	5.023	1.868	4.120***	3.839**
	(1.853)	(2.111)	(3.424)	(3.757)	(1.282)	(1.532)
Constant	-22.19***	-15.00***	-21.65***	-18.76***	-26.77***	-24.02***
	(2.476)	(2.280)	(5.558)	(5.575)	(1.922)	(1.592)
R^2	0.037	0.441	0.004	0.322	0.034	0.649
PANEL D: Consumer Prices - Freq. alone						
$Freq/\bar{F}$	-7.453***	-12.27***	-23.68***	-30.84***	-5.134***	-6.823***
	(2.799)	(1.467)	(7.510)	(6.122)	(1.401)	(0.923)
Constant	-9.170***	8.599***	7.053	30.76***	-17.52***	-9.996***
	(2.691)	(2.430)	(6.920)	(9.680)	(1.386)	(1.476)
R^2	0.188	0.716	0.257	0.575	0.151	0.784
Observations	223	223	223	223	223	223

Note: This table reports OLS results of a model (equation (13)) relating product-specific $CIR_T^{P_j}$ (expressed in %) to the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$ and OLS results of a model (equation (14)) relating product-specific $CIR_T^{P_j}$ (expressed in %) to the ratio of the product-level frequency over its average $Freq/\bar{F}$. Product fixed effects are defined at the 2-digit level for both CPI and PPI products (i.e. 38 product fixed effects for the CPI, and 24 in the case of the PPI). Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table A.6: Baseline OLS Regression Results : "Constrained" Specification - 24-month horizon

Identification	Cholesky		Cholesky		High-Freq. IV	
Long-run Restriction	Yes		No		Yes	
Product FE	No	Yes	No	Yes	No	Yes
PANEL A: PRODUCER PRICES						
Kurt/Freq	0.141**	0.0761*	0.304**	0.132	0.0763**	0.0620**
	(0.0577)	(0.0442)	(0.137)	(0.0966)	(0.0330)	(0.0299)
Constant	-15.31***	-6.978***	-22.41**	-14.86***	-15.63***	-12.41**
	(3.649)	(2.580)	(8.686)	(3.799)	(2.009)	(4.910)
Observations	118	118	118	118	118	118
R^2	0.067	0.476	0.057	0.463	0.065	0.403
PANEL B: CONSUMER PRICES						
Kurt/Freq	-0.00926	0.0263**	0.00510	0.0511**	0.00182	0.0192**
	(0.0107)	(0.0112)	(0.0216)	(0.0238)	(0.00697)	(0.00761)
Constant	-6.332***	-1.102	-7.627**	-5.092**	-11.76***	-9.189***
	(1.479)	(1.148)	(3.101)	(2.029)	(1.060)	(0.572)
Observations	223	223	223	223	223	223
R^2	0.004	0.427	0.000	0.326	0.000	0.713

Note: this table reports results of OLS regressions (equation (13)) where the dependent variable is the product-specific $CIR_T^{P_j}$ (calculated for the horizon T=24 months, and expressed in %) and the right-hand-side variable is the ratio $Kurt/freq$. Each observation corresponds to a disaggregate CPI or PPI product. For CPI, the level of disaggregation is 5 digit-level of the ECOICOP classification (ie. '01.1.1.1') whereas for PPI, the product level is the 4-digit level of the NACE rev2 classification of sectors. PPI covers the manufacturing sectors whereas CPI covers about 60% of the whole French CPI (main products excluded are rents, cars, utilities like electricity). Product fixed effects are defined at the 2-digit level for both CPI and PPI products (i.e. 38 product fixed effects for the CPI, and 24 in the case of the PPI). Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table A.7: Regression Results - "Unconstrained" Specification - 24-month horizon

Identification	Cholesky		Cholesky		High-Freq. IV	
Long-run Restriction	Yes		No		Yes	
Product FE	No	Yes	No	Yes	No	Yes
PANEL A: PRODUCER PRICES						
$Freq/\bar{F}$	-5.284**	-2.862	-12.52**	-5.056	-3.038**	-2.165*
	(2.406)	(2.055)	(6.226)	(5.549)	(1.362)	(1.227)
$Kurt/\bar{K}$	6.869**	4.786	14.26*	12.29	3.439*	2.208
	(3.436)	(3.101)	(8.217)	(7.601)	(1.786)	(1.862)
Constant	-10.76***	-6.321*	-10.92	-17.26**	-12.71***	-10.47**
	(3.764)	(3.362)	(8.839)	(7.317)	(1.726)	(4.791)
Observations	118	118	118	118	118	118
R^2	0.169	0.499	0.167	0.481	0.177	0.436
PANEL B: CONSUMER PRICES						
$Freq/\bar{F}$	-4.553**	-8.613***	-12.43***	-17.80***	-2.032**	-3.338***
	(2.149)	(1.135)	(4.400)	(3.281)	(0.815)	(0.491)
$Kurt/\bar{K}$	3.223**	2.147*	2.045	-1.007	2.371***	2.090***
	(1.291)	(1.275)	(1.802)	(1.920)	(0.842)	(0.789)
Constant	-5.837**	11.16***	3.219	25.02***	-11.93***	-5.280***
	(2.624)	(2.504)	(4.787)	(5.839)	(1.328)	(1.107)
Observations	223	223	223	223	223	223
R^2	0.161	0.700	0.254	0.607	0.085	0.803

Note: this table reports results of OLS regressions (equation (14)) where the dependent variable is the product-specific $CIR_T^{P_j}$ (calculated for the horizon T=24 months, and expressed in %) and the right-hand-side variables are the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$. Product fixed effects are defined at the 2-digit level for both CPI and PPI products (i.e. 38 product fixed effects for the CPI, and 24 in the case of the PPI). Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table A.8: Baseline OLS Regression Results : "Constrained" Specification - 48-month horizon

Identification	Cholesky		Cholesky		High-Freq. IV	
Long-run Restriction	Yes		No		Yes	
Product FE	No	Yes	No	Yes	No	Yes
PANEL A: PRODUCER PRICES						
Kurt/Freq	0.222***	0.127**	0.788**	0.320	0.208***	0.152**
	(0.0795)	(0.0577)	(0.373)	(0.269)	(0.0727)	(0.0635)
Constant	-38.74***	-28.64***	-63.41***	-56.04***	-46.91***	-40.34***
	(4.848)	(3.144)	(23.63)	(11.20)	(4.361)	(7.392)
Observations	118	118	118	118	118	118
R^2	0.099	0.548	0.052	0.457	0.100	0.419
PANEL B: CONSUMER PRICES						
Kurt/Freq	-0.000340	0.0494***	0.0496	0.136**	0.0172	0.0387***
	(0.0172)	(0.0168)	(0.0606)	(0.0663)	(0.0122)	(0.0134)
Constant	-27.12***	-22.87***	-31.63***	-36.74***	-35.97***	-34.58***
	(2.146)	(1.251)	(8.891)	(7.628)	(1.750)	(0.944)
Observations	223	223	223	223	223	223
R^2	0.000	0.454	0.003	0.333	0.009	0.643

Note: this table reports results of OLS regressions (equation (13)) where the dependent variable is the product-specific $CIR_T^{P_j}$ (calculated for the horizon T=48 months, and expressed in %) and the right-hand-side variable is the ratio $Kurt/freq$. Each observation corresponds to a disaggregate CPI or PPI product. For CPI, the level of disaggregation is 5 digit-level of the ECOICOP classification (ie. '01.1.1.1') whereas for PPI, the product level is the 4-digit level of the NACE rev2 classification of sectors. PPI covers the manufacturing sectors whereas CPI covers about 60% of the whole French CPI (main products excluded are rents, cars, utilities like electricity). Product fixed effects are defined at the 2-digit level for both CPI and PPI products (i.e. 38 product fixed effects for the CPI, and 24 in the case of the PPI). Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table A.9: Regression Results - “Unconstrained” Specification - 48-month horizon

Identification	Cholesky		Cholesky		High-Freq. IV	
Long-run Restriction	Yes		No		Yes	
Product FE	No	Yes	No	Yes	No	Yes
PANEL A: PRODUCER PRICES						
$Freq/\bar{F}$	-7.948**	-3.895	-33.08*	-11.52	-6.667**	-3.711
	(3.393)	(2.523)	(17.65)	(15.73)	(3.320)	(3.104)
$Kurt/\bar{K}$	9.160**	5.866	34.86	31.95	7.151*	3.654
	(4.521)	(3.968)	(22.02)	(20.91)	(3.713)	(3.827)
Constant	-30.28***	-26.23***	-30.85	-64.25***	-38.35***	-35.09***
	(4.707)	(3.988)	(23.51)	(20.66)	(3.775)	(6.830)
Observations	118	118	118	118	118	118
R^2	0.217	0.565	0.154	0.471	0.175	0.421
PANEL B: CONSUMER PRICES						
$Freq/\bar{F}$	-8.086***	-13.23***	-35.47***	-45.14***	-5.689***	-7.191***
	(3.098)	(1.686)	(11.06)	(9.409)	(1.530)	(1.087)
$Kurt/\bar{K}$	5.669***	3.676**	1.578	-7.284	4.129***	3.153**
	(1.942)	(1.788)	(6.047)	(5.400)	(1.299)	(1.270)
Constant	-24.74***	-4.089	6.738	44.08**	-32.85***	-25.04***
	(4.055)	(3.481)	(12.79)	(16.97)	(2.345)	(2.158)
Observations	223	223	223	223	223	223
R^2	0.211	0.719	0.241	0.545	0.179	0.774

Note: this table reports results of OLS regressions (equation (14)) where the dependent variable is the product-specific $CIR_T^{P_j}$ (calculated for the horizon T=48 months, and expressed in %) and the right-hand-side variables are the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$. Product fixed effects are defined at the 2-digit level for both CPI and PPI products (i.e. 38 product fixed effects for the CPI, and 24 in the case of the PPI). Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table A.10: Regression Results: Kurtosis Measurement - Heterogeneity

	PRODUCER PRICES			CONSUMER PRICES		
Identification	Cholesky	Cholesky	High-Freq. IV	Cholesky	Cholesky	High-Freq. IV
Long-run Restriction	Yes	No	Yes	Yes	No	Yes
PANEL A: Constrained model						
Kurt/Freq	0.222***	0.511*	0.205***	-0.00105	0.0306	0.0114
	(0.0801)	(0.269)	(0.0722)	(0.0135)	(0.0364)	(0.00972)
Constant	-26.47***	-36.24**	-32.78***	-16.56***	-18.57***	-23.38***
	(4.076)	(13.94)	(3.511)	(1.562)	(4.593)	(1.278)
R^2	0.078	0.036	0.091	0.000	0.002	0.004
PANEL B: Unconstrained model						
$Freq/\bar{F}$	-7.305**	-22.38*	-6.200**	-7.300**	-23.55***	-5.017***
	(3.206)	(11.77)	(2.856)	(2.843)	(7.568)	(1.419)
$Kurt/\bar{K}$	4.097**	2.870	2.951*	3.998***	3.252	3.045***
	(1.655)	(5.376)	(1.525)	(1.442)	(3.818)	(1.020)
Constant	-15.75***	0.560	-22.61***	-13.32***	3.676	-20.68***
	(2.366)	(8.067)	(2.055)	(3.485)	(8.574)	(1.996)
R^2	0.194	0.146	0.189	0.207	0.258	0.169
Observations	118	118	118	223	223	223

Note: This table reports OLS results of the constrained model (equation (13)) relating product-specific $CIR_T^{P_j}$ (expressed in %) to the ratio $Kurt/freq$ and OLS results of the unconstrained model (equation (14)) relating product-specific $CIR_T^{P_j}$ (expressed in %) to the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$. The measure of kurtosis takes into account for possible product heterogeneity following the methodology in Alvarez, Lippi, and Oskolkov (2022) and using $S = 5$ (see Appendix D). Product-fixed effects are not included. Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table A.11: Regression Results: 2-year German Bond - High-Frequency IV

	PRODUCER PRICES		*CONSUMER PRICES*	
Long-run Restriction	Yes		Yes	
Sector FE	No	Yes	No	Yes
PANEL A: Constrained model				
Kurt/Freq	0.255***	0.123*	0.0191***	0.0114
	(0.0765)	(0.0704)	(0.00603)	(0.00710)
Constant	-32.05***	-21.32***	-15.51***	-17.70***
	(4.607)	(3.599)	(0.987)	(1.083)
R^2	0.117	0.332	0.001	0.435
PANEL B: Unconstrained model				
$Freq/\bar{F}$	-6.550**	-1.951	-4.635***	-4.929***
	(2.974)	(3.040)	(0.851)	(0.811)
$Kurt/\bar{K}$	7.542*	3.040	0.790	-0.421
	(4.174)	(4.880)	(0.572)	(0.834)
Constant	-21.96***	-17.86***	-9.936***	-9.317***
	(4.974)	(5.509)	(1.021)	(1.505)
R^2	0.133	0.324	0.343	0.546
Observations	118	118	223	223

Note: This table reports OLS results of the constrained model (equation (13)) relating product-specific $CIR_T^{P_j}$ (expressed in %) to the ratio $Kurt/freq$ and OLS results of the unconstrained model (equation (14)) relating product-specific $CIR_T^{P_j}$ (expressed in %) to the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$. CIR of prices are here calculated using the 2-year German bond rate as a policy rate and the model is identified using a high frequency instrument method. Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table A.12: Regression Results: No drift - Sectoral Average Inflation< 5%

	PRODUCER PRICES			*CONSUMER PRICES*		
Identification	Cholesky	Cholesky	High-Freq. IV	Cholesky	Cholesky	High-Freq. IV
Long-run Restriction	Yes	No	Yes	Yes	No	Yes
Product FE	No	No	No	No	No	No
PANEL A: Constrained model						
Kurt/Freq	0.193***	0.499**	0.189***	-0.00425	0.0507	0.0136
	(0.0734)	(0.245)	(0.0648)	(0.0154)	(0.0396)	(0.0111)
Constant	-26.76***	-37.59**	-34.24***	-16.03***	-23.33***	-24.00***
	(4.452)	(15.20)	(3.871)	(2.069)	(5.761)	(1.669)
R^2	0.100	0.059	0.111	0.000	0.009	0.007
PANEL B: Unconstrained model						
$Freq/\bar{F}$	-7.619**	-23.58**	-6.256**	-7.259**	-24.25***	-4.998***
	(3.170)	(11.65)	(2.845)	(2.841)	(7.252)	(1.401)
$Kurt/\bar{K}$	7.284*	17.85	7.197**	4.679***	4.606	3.799***
	(3.855)	(13.12)	(3.342)	(1.730)	(3.539)	(1.259)
Constant	-17.98***	-10.04	-26.90***	-13.84***	1.009	-21.54***
	(3.613)	(12.31)	(3.293)	(3.600)	(7.965)	(2.135)
R^2	0.254	0.208	0.205	0.214	0.311	0.181
Observations	116	116	116	214	214	214

Note: This table reports OLS results of the constrained model (equation (13)) relating product-specific $CIR_T^{P_j}$ (expressed in %) to the ratio $Kurt/freq$ and OLS results of the unconstrained model (equation (14)) relating product-specific $CIR_T^{P_j}$ (expressed in %) to the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$. We remove products for which the average annual inflation is above 5% (in absolute values) over the sample period. Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table A.13: Regression Results: Aggregate Sectoral Fixed Effects

	PRODUCER PRICES			*CONSUMER PRICES*		
Identification	Cholesky	Cholesky	High-Freq. IV	Cholesky	Cholesky	High-Freq. IV
Long-run Restriction	Yes	No	Yes	Yes	No	Yes
PANEL A: Constrained model						
Kurt/Freq	0.227***	0.580**	0.215***	-0.0215	-0.0527	0.00200
	(0.0712)	(0.234)	(0.0671)	(0.0184)	(0.0430)	(0.0201)
Constant	-28.95***	-40.43***	-37.77***	-8.687***	-15.96***	-20.09***
	(4.180)	(13.99)	(4.856)	(0.926)	(3.933)	(0.910)
R^2	0.255	0.228	0.213	0.407	0.322	0.424
PANEL B: Unconstrained model						
$Freq/\bar{F}$	-8.099***	-23.18**	-6.937**	-7.855***	-19.66**	-3.910**
	(2.563)	(9.943)	(2.880)	(2.497)	(8.290)	(1.592)
$Kurt/\bar{K}$	13.75**	46.93**	11.53***	3.709**	3.206	3.760*
	(5.439)	(18.48)	(4.329)	(1.457)	(3.012)	(1.967)
Constant	-26.10***	-44.83**	-33.80***	-1.930	7.300	-18.24***
	(5.609)	(18.61)	(5.211)	(4.022)	(12.56)	(2.763)
R^2	0.400	0.355	0.333	0.498	0.390	0.481
Observations	118	118	118	223	223	223

Note: This table reports OLS results of the constrained model (equation (13)) relating product-specific $CIR_T^{P_j}$ (expressed in %) to the ratio $Kurt/freq$ and OLS results of the unconstrained model (equation (14)) relating product-specific $CIR_T^{P_j}$ (expressed in %) to the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$. For CPI, 12 product fixed effects are included corresponding to COICOP 1-digit product categories; for PPI, 6 product fixed effects are included (capital goods, consumer goods (food), consumer goods (durable), consumer goods (other), intermediate goods, energy). Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table A.14: Regression Results: Product level inflation - Production volatility

Identification	Cholesky		Cholesky		High-Freq. IV	
Long-run Restriction	Yes		No		Yes	
Product FE	No	Yes	No	Yes	No	Yes
PANEL A: PRODUCER PRICES						
Kurt/Freq	0.184***	0.100*	0.431**	0.171	0.179***	0.134**
	(0.0594)	(0.0533)	(0.183)	(0.169)	(0.0573)	(0.0542)
Production volatility	0.0400	0.449	0.921	1.219	-0.0100	0.0562
	(0.151)	(0.277)	(0.605)	(1.073)	(0.155)	(0.264)
Average inflation	-6.209*	-4.401	-27.76**	-21.08**	-2.823	0.137
	(3.322)	(2.714)	(10.94)	(10.27)	(2.658)	(2.324)
Constant	-20.41***	-12.88	-16.72	-2.428	-30.36***	-28.04***
	(3.364)	(8.025)	(12.66)	(24.46)	(3.514)	(8.201)
R^2	0.225	0.580	0.282	0.549	0.147	0.452
Observations	118	118	118	118	118	118
PANEL B: CONSUMER PRICES						
Kurt/Freq	-0.00234	0.0415**	0.0604	0.108**	0.0161	0.0324**
	(0.0174)	(0.0161)	(0.0443)	(0.0455)	(0.0126)	(0.0126)
Average inflation	0.0165	0.0220	-4.328***	-3.026	-0.477	0.0810
	(0.389)	(0.459)	(1.386)	(1.880)	(0.340)	(0.299)
Constant	-16.43***	-11.76***	-17.80***	-16.45***	-23.64***	-21.71***
	(1.869)	(1.159)	(5.270)	(4.866)	(1.490)	(0.714)
R^2	0.000	0.439	0.066	0.345	0.016	0.649
Observations	223	223	223	223	223	223

Note: this table reports results of OLS regressions (equation (14)) where the dependent variable is the product-specific $CIR_T^{P_j}$ (calculated for the horizon T=36 months, and expressed in %) and the right-hand-side variables include the product-specific ratio $Kurt/freq$ but also, for CPI and PPI products, the average sectoral CPI or PPI inflation (calculated as mean of the year-on-year growth in sectoral price indices (source Insee) over the sample period) and for PPI, the volatility of the aggregate production (calculated as the standard deviation of the year-on-year growth in sectoral production indices (source Insee) over the sample period). Product fixed effects are defined at the 2-digit level for both CPI and PPI products (i.e. 38 product fixed effects for the CPI, and 24 in the case of the PPI). Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table A.15: OLS Regression Results - CIR of Output - PPI products

Identification	Cholesky		Cholesky		HFI	
Long-run Restriction	Yes		No		Yes	
Product FE	No	Yes	No	Yes	No	Yes
PANEL A: Constrained model						
Kurt/Freq	-0.373	-0.439	-0.144	-0.843	0.0446	-0.0200
	(0.296)	(0.343)	(1.009)	(1.146)	(0.243)	(0.329)
Constant	-23.67**	42.45	-20.59	169.4***	29.81***	154.4***
	(10.33)	(26.61)	(33.48)	(46.82)	(9.184)	(15.71)
R^2	0.027	0.240	0.000	0.201	0.000	0.187
PANEL B: Unconstrained model						
$Freq/\bar{F}$	8.191**	8.649**	-1.683	8.198	-0.181	1.809
	(3.420)	(3.468)	(11.39)	(9.728)	(4.283)	(5.671)
$Kurt/\bar{K}$	-14.76	-16.70	-13.20	-21.27	-9.610	-27.84
	(13.85)	(17.69)	(33.64)	(51.75)	(13.56)	(20.16)
Constant	-31.89**	35.20	-11.42	150.8***	41.37**	178.1***
	(16.08)	(31.22)	(39.24)	(50.90)	(16.18)	(22.24)
R^2	0.025	0.236	0.001	0.194	0.002	0.198
Observations	156	156	156	156	156	156

Note: this table reports results of OLS regressions testing equation (5) where the dependent variable is the product-specific $CIR_T^{Y_j}$ (expressed in %) and the right-hand-side variable are the ratio $Kurt/freq$ (Panel A) or the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$ (Panel B). Each observation corresponds to a disaggregate PPI product, the product level is the 4-digit level of the NACE rev2 classification of sectors. PPI covers the manufacturing sectors. Product fixed effects are defined at the 2-digit level (24). Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table A.16: OLS Regression Results: Moments of Price Durations (Alvarez et al. 2016)

Identification	Cholesky		Cholesky		High-Freq. IV	
Long-run Restriction	Yes		No		Yes	
Product FE	No	Yes	No	Yes	No	Yes
PANEL A: Producer prices - Constrained model						
$E(d)\,(1+CV(d)^2)$	2.288***	1.347*	6.310**	2.428	1.873**	1.261*
	(0.830)	(0.713)	(2.960)	(2.824)	(0.724)	(0.690)
Constant	-43.58***	-30.43***	-86.89**	-56.01	-46.03***	-38.04***
	(10.22)	(9.411)	(36.10)	(35.96)	(8.822)	(11.00)
R^2	0.175	0.553	0.116	0.472	0.162	0.463
PANEL B: Producer prices - Unconstrained model						
$E(d)$	2.529***	1.474**	6.351**	2.838	2.110***	1.493**
	(0.755)	(0.594)	(2.676)	(2.278)	(0.635)	(0.601)
$CV(d)^2$	1.742	1.122	6.217	1.702	1.336	0.849
	(1.235)	(1.124)	(4.407)	(4.561)	(1.067)	(1.035)
Constant	-42.31***	-29.98***	-86.67**	-54.56	-44.77***	-37.22***
	(10.88)	(10.24)	(38.53)	(39.07)	(9.478)	(12.03)
R^2	0.179	0.553	0.116	0.472	0.167	0.466
Observations	118	118	118	118	118	118
PANEL C: Consumer prices - Constrained model						
$E(d)\,(1+CV(d)^2)$	0.923*	2.113***	2.361	4.645**	0.825**	1.312***
	(0.535)	(0.661)	(1.617)	(1.992)	(0.353)	(0.358)
Constant	-27.63***	-30.85***	-44.76**	-61.96***	-32.46***	-33.26***
	(7.073)	(6.484)	(21.50)	(20.11)	(4.812)	(3.563)
R^2	0.038	0.520	0.034	0.386	0.052	0.696
PANEL D: Consumer prices - Unconstrained model						
$E(d)$	0.684**	1.688***	2.218**	3.683***	0.783***	1.150***
	(0.331)	(0.523)	(1.038)	(1.371)	(0.264)	(0.336)
$CV(d)^2$	2.468	4.033**	3.278	8.981	1.098	2.042*
	(1.952)	(1.888)	(5.957)	(5.933)	(1.139)	(1.078)
Constant	-32.84***	-37.64***	-47.86	-77.31**	-33.38***	-35.84***
	(11.30)	(10.00)	(35.48)	(32.79)	(7.110)	(5.488)
R^2	0.051	0.532	0.035	0.394	0.053	0.699
Observations	219	219	219	219	219	219

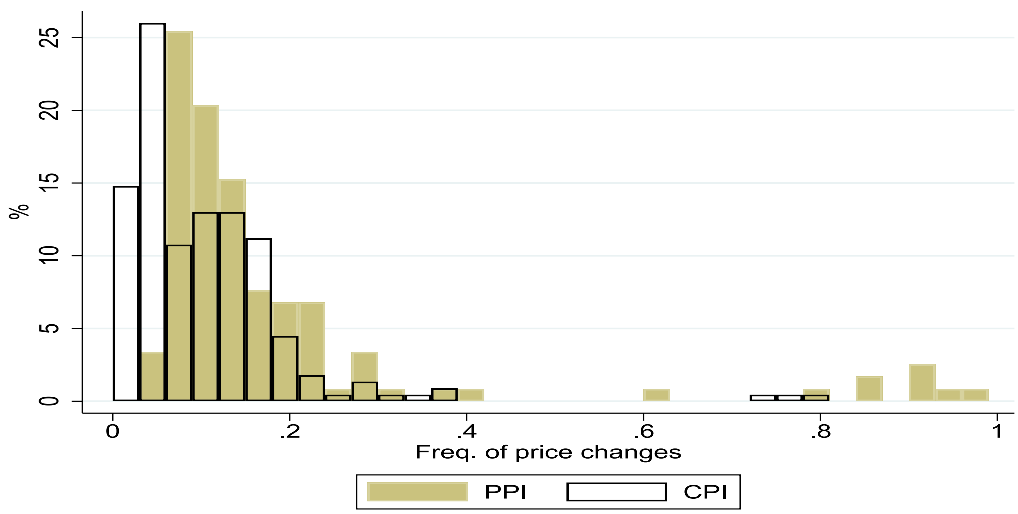
Note: This table reports results of OLS regressions relating product-specific $CIR_T^{P_j}$ (expressed in %) to the product $E(d)\left(1+CV(d)^2\right)$ where $E(d)$ is the average price duration calculated at the product level and $CV(d)$ is the coefficient of variation of price durations (Panels A and C), and results of OLS regressions relating product-specific $CIR_T^{P_j}$ (expressed in %) to $E(d)$ and $CV(d)^2$ (Panels B and D). Product fixed effects are defined at the 2-digit level for both CPI and PPI products (i.e. 38 product fixed effects for the CPI, and 24 in the case of the PPI). Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

SUPPLEMENTARY APPENDIX

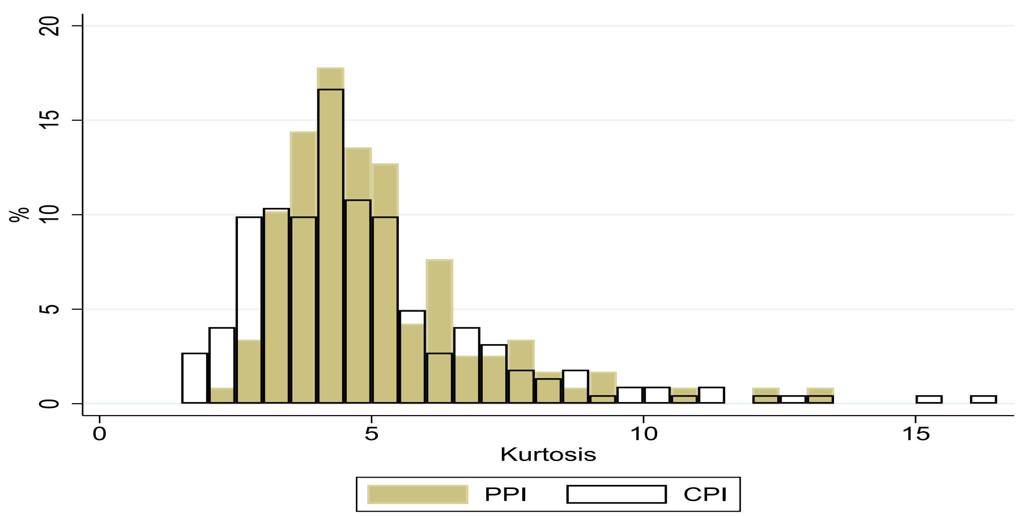
Additional figures

Figure B.1: Cross-sector Distribution of Frequency and Kurtosis of Price Changes (CPI-PPI)

(a) Frequency

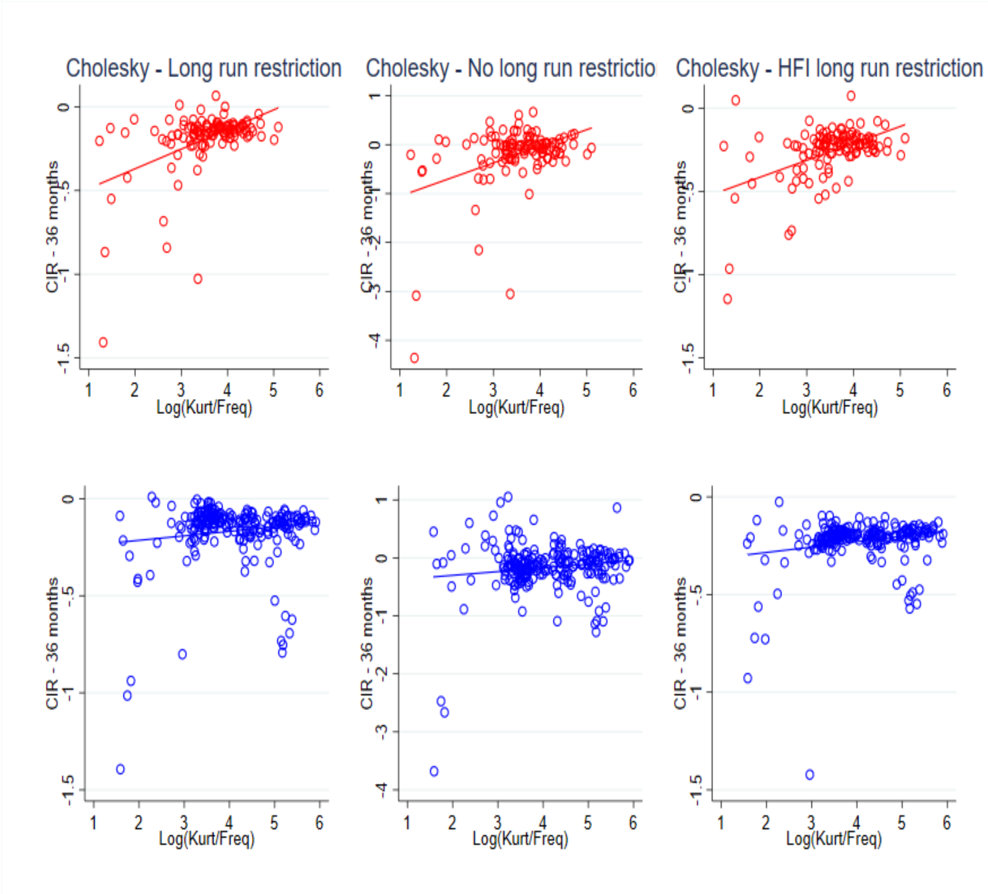


(b) Kurtosis



Note: histograms report the distribution of frequency and kurtosis separately for 118 PPI products and 227 CPI products.

Figure B.2: Correlation CIR^P - Log ratio $\frac{Kurt}{Freq}$



Note: the figure plots for each of the three FAVAR specifications the product-specific CIR (at the horizon 36 months) and the log of the ratio kurtosis over frequency of price changes. The top panel (red dots) reports results for PPI products whereas the bottom panel (blue dots) reports results for CPI products.

Measurement Error

This appendix assesses the impact of (one form of) measurement errors on the micro moments of price adjustment and their ratio $Kurt/Freq$.

Assume measurement errors are of the following type: for a given store, measurement errors materialize at some points by extra spurious price changes, and these spurious price changes are small. Such patterns of error is plausible (as discussed in Alvarez, Le Bihan, and Lippi (2016)), both for CPI data because small coding error can stay undetected by the error checking procedures of the statistical institute, and for scanner data as the price in typically computed as the ratio of value purchased to quantity sold (and the numerator can vary reflecting e.g. coupons). These spurious price changes will increase both the measured Kurtosis, as well as the measured Frequency of price changes - with the size of the bias being a function of the fraction of spurious price changes. However, as is formally shown below, such measurement errors will leave ratio Kurtosis/Frequency unchanged. As a result, not only theory indicates that the ratio Kurtosis/Frequency is the relevant covariate, but it is also the case that this ratio should be more robust to measurement errors than each of the moments taken separately.

Formally, let $N_{\Delta p}$ be the number of "true" price changes per period (i.e. the frequency of price changes). Assume Δp, the price changes, have mean zero, variance $Var(\Delta p) = \sigma^2_{\Delta p}$ and Kurtosis $Kurt(\Delta p) = m_{4,\Delta p}/\sigma^4_{\Delta p}$, where $m_{4,\Delta p}$ is the fourth moment of variable Δp . Let N_e denote the number of spurious price changes per unit of time. Assume that spurious price changes, denoted e, have mean zero and variance $Var(e) = \sigma^2_e$, and kurtosis $Kurt(e) = m_{4,e}/\sigma^4_e$. Aume spurious and true price changes to be statistically independent. Then the observed (measured) frequency of price changes will be $\tilde{N} = N_{\Delta p} + N_e$. The distribution of the observed price changes, denoted $\tilde{\Delta} p$'s , will have mean zero and its Kurtosis will be

$$Kurt(\tilde{\Delta} p) = \frac{\theta Kurt(\Delta p)\sigma^4_{\Delta p} + (1-\theta) Kurt(e)\sigma^4_e}{\left(\theta\sigma^2_{\Delta p} + (1-\theta)\sigma^2_e\right)^2}$$

with $\theta \equiv \frac{N_{\Delta p}}{\tilde{N}}$ the fraction of "true" price changes. We consider the case of arbitrarily small measurement errors . From the above it results that $\lim_{\sigma^2_e \to 0} Kurt(\tilde{\Delta} p) = \frac{Kurt(\Delta p)}{\theta}$. Then we have $\lim_{\sigma^2_e \to 0} \frac{Kurt(\tilde{\Delta} p)}{\tilde{N}} = \frac{Kurt(\Delta p)}{N_{\Delta p}}$. Thus, the ratio Kurtosis over Frequency is unaffected by these presence of small measurement error.

Further robustness regression results

Robustness: Removing Extreme Values of CIR^P, $Freq$, $Kurt$ or $Kurt/Freq$

Another robustness exercise consists of checking whether our main results are driven by some products for which the cumulative response of prices, frequency or kurtosis of price changes, is either extremely low or extremely high. For that we define 4 sub-samples in which we remove 5% of products corresponding to the 2.5% largest or the 2.5% smallest values for: (i) the CIR^P, (ii) ratio kurtosis over frequency, (iii) kurtosis of non-zero price changes or (iv) frequency of price changes.[42] We run our baseline regressions on each of these subsamples and results are reported in tables Table B.1 and Table B.2 in the Appendix.

For PPI products, removing products with extreme values does not alter our baseline conclusions: in most cases, the slope coefficient associated with the ratio $Kurt/Freq$ is positive and significantly different from zero, and estimated coefficients are quite close to the ones estimated in our baseline exercise. We find a strongest relationship when we exclude extreme values of kurtosis. Similarly, in unconstrained regressions, results are fully in line with the ones using the full sample of products.

For CPI products, results obtained when removing 'extreme' products are similar with baseline results and the ratio $Kurt/Freq$ is not statistically different from zero. In the unconstrained version of the model, kurtosis is positive and significantly different from 0 in most cases but the estimated parameter associated with frequency is non-significantly different from 0 in several cases.

[42] For CPI, 10 different products are excluded in each subsample whereas for PPI 6 different products are excluded.

Table B.1: Regression Results: Outliers - Constrained Model

	PRODUCER PRICES			*CONSUMER PRICES*		
Identification	Cholesky	Cholesky	High-Freq. IV	Cholesky	Cholesky	High-Freq. IV
Long-run Restriction	Yes	No	Yes	Yes	No	Yes
Product FE	No	No	No	No	No	No
PANEL A: CIR						
Kurt/Freq	0.120***	0.224**	0.126***	-0.0159	-0.00518	0.000885
	(0.0427)	(0.109)	(0.0384)	(0.0105)	(0.0224)	(0.00675)
Constant	-22.44***	-22.17***	-30.41***	-13.58***	-13.66***	-21.54***
	(2.672)	(7.196)	(2.283)	(0.879)	(2.771)	(0.655)
R^2	0.093	0.037	0.110	0.014	0.000	0.000
PANEL B: Ratio						
Kurt/Freq	0.193***	0.339**	0.184***	-0.0244	-0.00930	0.00360
	(0.0553)	(0.156)	(0.0547)	(0.0162)	(0.0406)	(0.0122)
Constant	-25.41***	-27.34***	-32.27***	-13.61***	-13.59***	-22.56***
	(3.433)	(9.998)	(3.099)	(1.500)	(4.569)	(1.527)
R^2	0.097	0.030	0.113	0.016	0.000	0.000
PANEL C: Kurtosis						
Kurt/Freq	0.282***	0.755**	0.257***	-0.00825	0.0121	0.0103
	(0.0886)	(0.303)	(0.0762)	(0.0173)	(0.0423)	(0.0122)
Constant	-31.08***	-51.06***	-36.96***	-16.13***	-19.19***	-23.84***
	(5.173)	(17.92)	(4.323)	(2.106)	(6.052)	(1.693)
R^2	0.128	0.080	0.147	0.001	0.000	0.004
PANEL D: Frequency						
Kurt/Freq	0.197**	0.619**	0.214***	-0.0155	-0.0165	0.00588
	(0.0803)	(0.276)	(0.0711)	(0.0109)	(0.0287)	(0.00955)
Constant	-27.28***	-45.96***	-35.44***	-13.35***	-10.35***	-22.13***
	(4.859)	(16.89)	(4.017)	(1.057)	(3.286)	(1.362)
R^2	0.082	0.067	0.133	0.011	0.002	0.002
Observations	112	112	112	213	213	213

Note: this table reports results of OLS regressions (equation (13)) for PPI products relating the product-specific $CIR_T^{P_j}$ (expressed in %) to the ratio $Kurt/freq$. For each of the 4 regressions, we remove products with "extreme" values of CIR (Panel A); ratio $Kurt/Freq$ (Panel B), kurtosis (Panel C), frequency of price changes (Panel D). "Extreme values" are defined as products below the 2.5th percentile or above the 97.5th percentile of the distribution of each statistic (10 products for CPI and 6 for PPI). Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table B.2: Regression Results: Outliers - Unconstrained

	PRODUCER PRICES			*CONSUMER PRICES*		
Identification	Cholesky	Cholesky	High-Freq. IV	Cholesky	Cholesky	High-Freq. IV
Long-run Restriction	Yes	No	Yes	Yes	No	Yes
Product FE	No	No	No	No	No	No
PANEL A: CIR						
$Freq/\bar{F}$	-3.486**	-5.952**	-3.897***	2.057	-0.762	-1.620
	(1.585)	(2.970)	(1.388)	(1.620)	(3.695)	(1.416)
$Kurt/\bar{K}$	2.883	5.554	4.452*	4.727***	1.712	2.764***
	(2.059)	(6.260)	(2.404)	(1.605)	(2.590)	(0.667)
Constant	-16.71***	-12.14	-25.71***	-21.70***	-15.15***	-22.76***
	(2.591)	(8.174)	(2.863)	(2.703)	(5.147)	(1.729)
R^2	0.105	0.036	0.133	0.076	0.002	0.086
PANEL B: Ratio						
$Freq/\bar{F}$	-3.932**	-6.477*	-3.474	-1.832	-12.97	-2.963**
	(1.805)	(3.471)	(2.215)	(3.805)	(9.849)	(1.453)
$Kurt/\bar{K}$	7.954**	11.80	6.120*	7.518***	4.381	5.366***
	(3.710)	(11.09)	(3.289)	(1.707)	(5.325)	(1.532)
Constant	-21.49***	-18.70	-27.36***	-21.37***	-6.719	-24.76***
	(4.337)	(14.04)	(3.263)			
R^2	0.085	0.022	0.080	0.071	0.069	0.076
PANEL C: Kurtosis						
$Freq/\bar{F}$	-7.334**	-22.41**	-6.206**	-7.886***	-26.38***	-5.602***
	(3.079)	(11.29)	(2.820)	(2.812)	(7.067)	(1.302)
$Kurt/\bar{K}$	16.25**	47.34**	12.35**	10.49***	3.434	7.042***
	(6.322)	(21.13)	(4.937)	(2.268)	(6.538)	(2.021)
Constant	-27.67***	-42.82**	-31.94***	-19.17***	4.334	-24.19***
	(6.068)	(20.58)	(4.441)	(3.835)	(9.826)	(2.654)
R^2	0.235	0.185	0.221	0.254	0.309	0.212
PANEL D: Frequency						
$Freq/\bar{F}$	-9.007*	-35.41**	-10.30***	3.269*	4.277	-0.819
	(4.644)	(15.89)	(3.225)	(1.684)	(3.852)	(1.614)
$Kurt/\bar{K}$	7.728*	19.20	6.470**	4.608***	2.669	3.468***
	(3.957)	(13.74)	(3.000)	(1.624)	(3.185)	(1.116)
Constant	-18.22***	-5.917	-23.19***	-22.38***	-18.41***	-24.39***
	(4.688)	(16.94)	(3.175)	(2.700)	(5.574)	(1.709)
R^2	0.213	0.253	0.360	0.080	0.009	0.037
Observations	112	112	112	213	213	213

Note: This table reports results of OLS regressions (equation (14)) relating the product-specific $CIR_T^{P_j}$ (expressed in %) to the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$. For each of the 4 regressions, we remove products with "extreme" values of CIR (Panel A); ratio $Kurt/Freq$ (Panel B), kurtosis (Panel C), frequency of price changes (Panel D). "Extreme values" are defined as products below the 2.5th percentile or above the 97.5th percentile of the distribution of each statistic. Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Robustness: A FAVAR with only PPI Sectoral Data

In this exercise, we carry out the analysis removing the CPI sectoral data from the FAVAR - and thus producing cross-sectoral regressions only for PPI. The motivation for carrying such an exercise is related to concerns with the CPI data: even if one restricts interest to the cross sectoral-regressions related to PPI data only, they are still potentially affected by the properties of CPI data, since CPI data will influence the computation of the factors and hence of sectoral IRFs for PPI as well.

We carried out the analysis with PPI only both under the standard identification case, as well as in the case using the German 2-year bond yields, with HFI identification. Results are reported in Appendix Table B.3 and are in all cases highly similar to the ones obtained in our baseline regressions.

Table B.3: Regression Results: FAVAR PPI only - Euribor vs German 2-year Bond

Identification	Cholesky		Cholesky		High-Freq. IV	
Long-run Restriction	Yes		No		Yes	
Product FE	No	Yes	No	Yes	No	Yes
PANEL A: Constrained model						
Kurt/Freq	0.101***	0.0543**	0.711**	0.321	0.340***	0.192**
	(0.0288)	(0.0223)	(0.326)	(0.253)	(0.0939)	(0.0744)
Constant	-22.81***	-17.88***	-49.38**	-28.81**	-38.40***	-23.70***
	(1.749)	(1.686)	(21.19)	(12.33)	(5.511)	(4.714)
R^2	0.143	0.497	0.050	0.404	0.160	0.508
PANEL B: Unconstrained model						
$Freq/\bar{F}$	-3.016**	-1.427	-29.53**	-14.23	-9.948***	-4.590
	(1.190)	(1.180)	(14.07)	(14.28)	(3.597)	(3.451)
$Kurt/\bar{K}$	4.767***	3.528**	39.56**	36.79*	12.50***	7.041
	(1.530)	(1.441)	(19.93)	(18.89)	(4.729)	(4.505)
Constant	-20.14***	-17.93***	-28.42	-39.48**	-26.14***	-19.35***
	(1.650)	(1.501)	(22.34)	(18.51)	(5.159)	(4.402)
R^2	0.243	0.517	0.156	0.431	0.241	0.512
Observations	118	118	118	118	118	118

Note: This table reports OLS results of the constrained model (equation (13)) relating product-specific $CIR_T^{P_j}$ (expressed in %) to the ratio $Kurt/freq$ and OLS results of the unconstrained model (equation (14)) relating product-specific $CIR_T^{P_j}$ (expressed in %) to the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$. CIR are here calculated from a FAVAR model estimated using only PPI product-level series using both the Euribor (with no instrument) and the 2-year German bond rate as a policy rate, in this latter model, the model is identified using a high frequency instrument method. Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

More regression results using CIR^Y

Table B.4: Regression Results - CIR of Output - PPI products - Placebo Specification

Identification	Cholesky		Cholesky		High-Freq. IV	
Long-run Restriction	Yes		No		Yes	
Product FE	No	Yes	No	Yes	No	Yes
Kurt/Freq	-0.911**	-0.818*	-1.020	-1.102	0.0788	-0.0335
	(0.430)	(0.472)	(1.483)	(1.492)	(0.358)	(0.504)
Mean	18.22***	6.443	15.22	-4.582	6.720	7.794
	(6.250)	(8.149)	(20.11)	(24.96)	(10.71)	(10.57)
Skew.	-12.78	-11.62	-61.64	-39.11	8.915	4.278
	(18.58)	(24.51)	(54.73)	(75.98)	(21.30)	(29.96)
SD	-6.547	-11.17*	8.853	3.640	6.941	11.74
	(6.643)	(6.635)	(17.14)	(20.39)	(6.846)	(8.779)
Constant	9.957	96.99**	-48.58	153.6	-3.397	94.17**
	(33.86)	(37.43)	(96.34)	(107.6)	(33.29)	(46.92)
Observations	156	156	156	156	156	156
R^2	0.089	0.267	0.020	0.204	0.010	0.202

Note: this table reports results of OLS regressions testing equation (5) where the dependent variable is the product-specific $CIR_T^{Y_j}$ (expressed in %) and the right-hand-side variable are the ratio $Kurt/freq$ but also three other moments of the product-specific price change distribution: the average price change $Mean$, the skewness of price changes $Skewness$, and the standard deviation of price changes $StandardDev.$. Each observation corresponds to a disaggregate PPI product, the product level is the 4-digit level of the NACE rev2 classification of sectors. PPI covers the manufacturing sectors. Product fixed effects are defined at the 2-digit level (24). Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table B.5: Regression Results - CIR of Output - PPI products - Placebo Specification

Identification	Cholesky		Cholesky		High-Freq. IV	
Long-run Restriction	Yes		No		Yes	
Product FE	No	Yes	No	Yes	No	Yes
$Freq/\bar{F}$	10.72***	9.015**	6.106	5.540	1.345	4.557
	(3.505)	(3.905)	(9.814)	(10.88)	(4.456)	(5.459)
$Kurt/\bar{K}$	-52.46	-66.77**	-12.79	-2.117	0.430	-20.38
	(32.38)	(30.94)	(70.01)	(83.14)	(33.26)	(46.44)
Mean	12.66*	-0.951	7.906	-15.79	7.731	8.732
	(6.918)	(11.04)	(22.45)	(35.14)	(11.26)	(11.73)
Skew.	1.825	-9.391	-33.73	-15.11	5.394	-0.962
	(14.45)	(16.55)	(35.06)	(53.03)	(15.99)	(23.20)
SD	-15.67	-23.19**	9.957	6.587	7.020	8.063
	(12.46)	(11.35)	(29.62)	(33.17)	(13.57)	(16.76)
Constant	61.56	184.6**	-75.33	116.6	-3.914	122.6
	(82.37)	(81.05)	(191.5)	(222.3)	(89.80)	(114.2)
Observations	156	156	156	156	156	156
R^2	0.077	0.275	0.010	0.197	0.010	0.207

Note: this table reports results of OLS regressions testing equation (5) where the dependent variable is the product-specific $CIR_T^{Y_j}$ (expressed in %) and the right-hand-side variable are the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$ but also three other moments of the product-specific price change distribution: the average price change $Mean$, the skewness of price changes $Skewness$, and the standard deviation of price changes $StandardDev.$. Each observation corresponds to a disaggregate PPI product, the product level is the 4-digit level of the NACE rev2 classification of sectors. PPI covers the manufacturing sectors. Product fixed effects are defined at the 2-digit level (24). Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table B.6: Regression Results - CIR of Output - PPI products - Excluding 10% of sectors with highest positive CIR^Y

Identification	Cholesky		Cholesky		High-Freq. IV	
Long-run Restriction	Yes		No		Yes	
Product FE	No	Yes	No	Yes	No	Yes
PANEL A: Constrained model						
Kurt/Freq	-0.579**	-0.655**	-0.557	-1.016	-0.269**	-0.459***
	(0.279)	(0.302)	(0.933)	(0.927)	(0.125)	(0.154)
Constant	-28.28***	23.54	-39.89	161.2***	25.50***	5.644
	(9.874)	(15.09)	(31.73)	(46.25)	(7.122)	(8.758)
R^2	0.107	0.350	0.014	0.314	0.022	0.198
PANEL B: Unconstrained model						
$Freq/\bar{F}$	11.71***	9.881***	6.674	6.570	5.532	8.220*
	(2.846)	(3.152)	(9.602)	(8.429)	(3.451)	(4.270)
$Kurt/\bar{K}$	-1.418	-2.098	8.333	-31.28	-13.72	-29.25**
	(11.75)	(16.17)	(29.30)	(45.28)	(9.742)	(14.50)
Constant	-61.27***	-11.16	-77.11***	132.2***	23.00*	11.58
	(11.67)	(12.15)	(26.67)	(34.02)	(12.28)	(17.91)
R^2	0.064	0.301	0.003	0.292	0.023	0.204
Observations	140	140	140	140	140	140

Note: this table reports results of OLS regressions testing equation (5) where the dependent variable is the product-specific $CIR_T^{Y_j}$ (expressed in %) and the right-hand-side variable are the ratio $Kurt/freq$ (Panel A) or the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$ (Panel B). Each observation corresponds to a disaggregate PPI product, the product level is the 4-digit level of the NACE rev2 classification of sectors. PPI covers the manufacturing sectors. Product fixed effects are defined at the 2-digit level (24). We have excluded 10% of sectors with the highest positive CIR^Y in each specification. Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Robustness: regression results using a different HP filter value $\lambda = 1M$

Table B.7: OLS Regression Results : "Constrained" Specification - HP filter $\lambda = 10^6$

Identification	Cholesky		Cholesky		High-Freq. IV	
Long-run Restriction	Yes		No		Yes	
Product FE	No	Yes	No	Yes	No	Yes
PANEL A: PRODUCER PRICES						
Kurt/Freq	0.216***	0.124**	0.539**	0.234	0.189***	0.135**
	(0.0726)	(0.0524)	(0.233)	(0.165)	(0.0646)	(0.0555)
Constant	-28.96***	-19.78***	-43.04***	-33.91***	-34.08***	-27.29***
	(4.386)	(2.906)	(14.66)	(6.844)	(3.834)	(6.984)
Observations	118	118	118	118	118	118
R^2	0.114	0.561	0.064	0.478	0.110	0.452
PANEL B: CONSUMER PRICES						
Kurt/Freq	0.00326	0.0438***	0.0268	0.0915**	0.0125	0.0328***
	(0.0154)	(0.0153)	(0.0397)	(0.0432)	(0.0107)	(0.0118)
Constant	-17.59***	-14.36***	-19.71***	-21.85***	-23.78***	-21.61***
	(1.973)	(1.058)	(5.768)	(4.445)	(1.582)	(0.768)
Observations	223	223	223	223	223	223
R^2	0.000	0.437	0.002	0.347	0.006	0.649

Note: this table reports results of OLS regressions (equation (13)) where the dependent variable is the product-specific $CIR_T^{P_j}$ (calculated for the horizon T=36 months, and expressed in %) and the right-hand-side variable is the ratio $Kurt/freq$. Each observation corresponds to a disaggregate CPI or PPI product. For CPI, the level of disaggregation is 5 digit-level of the ECOICOP classification (ie. '01.1.1.1') whereas for PPI, the product level is the 4-digit level of the NACE rev2 classification of sectors. PPI covers the manufacturing sectors whereas CPI covers about 60% of the whole French CPI (main products excluded are rents, cars, utilities like electricity). Product fixed effects are defined at the 2-digit level for both CPI and PPI products (i.e. 38 product fixed effects for the CPI, and 24 in the case of the PPI). Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table B.8: Regression Results - "Unconstrained" Specification - HP filter $\lambda = 10^6$

Identification	Cholesky		Cholesky		High-Freq. IV	
Long-run Restriction	Yes		No		Yes	
Product FE	No	Yes	No	Yes	No	Yes
PANEL A: PRODUCER PRICES						
$Freq/\bar{F}$	-7.431**	-3.552	-21.36*	-7.623	-6.241**	-3.610
	(3.099)	(2.343)	(10.90)	(9.512)	(2.827)	(2.575)
$Kurt/\bar{K}$	8.708**	5.463	23.33*	20.11	7.065**	3.930
	(4.008)	(3.477)	(13.56)	(12.61)	(3.300)	(3.305)
Constant	-20.81***	-17.36***	-21.50	-37.54***	-26.68***	-23.04***
	(4.132)	(3.422)	(14.29)	(12.42)	(3.234)	(6.445)
Observations	118	118	118	118	118	118
R^2	0.233	0.576	0.170	0.492	0.205	0.462
PANEL B: CONSUMER PRICES						
$Freq/\bar{F}$	-8.079***	-12.49***	-23.61***	-30.74***	-4.989***	-6.636***
	(2.858)	(1.599)	(7.484)	(5.989)	(1.409)	(0.944)
$Kurt/\bar{K}$	4.706***	2.708*	1.931	-3.643	3.541***	2.687**
	(1.621)	(1.456)	(3.671)	(3.461)	(1.161)	(1.076)
Constant	-13.93***	3.967	4.380	31.97***	-21.21***	-12.69***
	(3.600)	(3.112)	(8.504)	(10.71)	(2.068)	(1.848)
Observations	223	223	223	223	223	223
R^2	0.247	0.726	0.256	0.582	0.176	0.793

Note: this table reports results of OLS regressions (equation (14)) where the dependent variable is the product-specific $CIR_T^{P_j}$ (calculated for the horizon T=36 months, and expressed in %) and the right-hand-side variables are the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$. Product fixed effects are defined at the 2-digit level for both CPI and PPI products (i.e. 38 product fixed effects for the CPI, and 24 in the case of the PPI). Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table B.9: Regression Results - Placebo Specification - HP filter $\lambda = 10^6$

Identification	Cholesky		Cholesky		High-Freq. IV	
Long-run Restriction	Yes		No		Yes	
Product FE	No	Yes	No	Yes	No	Yes
PANEL A: PRODUCER PRICES						
Kurt/Freq	0.201**	0.100	0.506*	0.209	0.188**	0.0987
	(0.0919)	(0.0716)	(0.294)	(0.254)	(0.0768)	(0.0666)
Mean	-1.381	-0.782	-10.56*	-4.271	-1.262	1.273
	(1.735)	(1.592)	(6.291)	(7.035)	(1.522)	(1.677)
Skewness	-3.019	-5.004	-17.24	-7.925	-2.207	-3.327
	(3.933)	(4.022)	(15.52)	(18.72)	(2.718)	(3.625)
SD	-0.951	0.976	-5.486	-3.173	-0.182	2.197
	(2.436)	(2.084)	(8.011)	(7.892)	(2.011)	(1.728)
Constant	-24.10***	-24.27**	-15.18	-16.60	-32.89***	-38.56***
	(9.063)	(11.01)	(28.38)	(40.53)	(7.834)	(11.33)
Observations	118	118	118	118	118	118
R^2	0.119	0.567	0.082	0.481	0.113	0.465
PANEL B: CONSUMER PRICES						
Kurt/Freq	-0.00992	0.0290	0.0921	0.160***	0.0116	0.0299**
	(0.0233)	(0.0222)	(0.0563)	(0.0604)	(0.0140)	(0.0146)
Mean	1.224**	1.675**	-1.157	0.376	0.132	0.759*
	(0.588)	(0.710)	(1.884)	(1.884)	(0.438)	(0.427)
Skewness	6.449**	3.852	18.69**	5.153	4.161**	3.868**
	(2.717)	(2.933)	(8.538)	(9.313)	(1.723)	(1.796)
SD	-0.527	0.240	3.290*	5.199**	-0.528	0.251
	(0.732)	(0.868)	(1.880)	(2.166)	(0.532)	(0.517)
Constant	-12.21*	-16.06**	-44.27**	-62.63***	-18.77***	-23.17***
	(7.268)	(6.776)	(18.76)	(18.11)	(4.394)	(4.001)
Observations	223	223	223	223	223	223
R^2	0.027	0.452	0.056	0.381	0.027	0.658

Note: this table reports results of OLS regressions (equation (13)) where the dependent variable is the product-specific $CIR_T^{P_j}$ (calculated for the horizon T=36 months and expressed in %) and the right-hand-side variables include the product-specific ratio $Kurt/freq$ but also three other moments of the product-specific price change distribution: the average price change $Mean$, the skewness of price changes $Skewness$, and the standard deviation of price changes $StandardDev.$. Product fixed effects are defined at the 2-digit level for both CPI and PPI products (i.e. 38 product fixed effects for the CPI, and 24 in the case of the PPI). Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Kurtosis Measurement with Unobserved Heterogeneity

The measure of Kurtosis is particularly sensitive to unobserved heterogeneity. Measured kurtosis is in particular known to suffer from an upward bias when a sample is composed of two (or more) sub-populations with different variances. To investigate the robustness of our results with respect to such a concern, we use an alternative measure of kurtosis derived along the lines of Alvarez, Lippi, and Oskolkov (2022). The assumption underlying this correction, is that within a given product category, there are several varieties (indexed by $i = 1, ..., I$) that are pooled. For instance, one could have various brands of soda, in the case the brand of soda is not collected by the statistical office, or not disclosed to the researcher. At a given date t, the price change for all varieties is driven by a common factor $\Delta\tilde{p}_t$, but the variance differs across varieties, according to a scaling factor b_i.

$$\Delta p_{it} = b_i \Delta\tilde{p}_t \text{ for } i \in I \text{ and } t \in T(i)$$

where $T(i)$ is the set of adjustment instances for variety i. Under the assumption that $\Delta\tilde{p}_t$ is serially uncorrelated, and some other general assumptions, Alvarez, Lippi, and Oskolkov (2022) show that the following property then holds:

$$Kurt(\Delta\tilde{p}_t) = Kurt(\Delta p_{it}) \frac{E[(\Delta p_{it}^2)]^2}{E[(\Delta p_{it}^2)(\Delta p_{is}^2)]} \text{ for } t \neq s$$

or equivalently

$$Kurt(\Delta\tilde{p}_t) = \frac{Kurt(\Delta p_{it})}{1 + corr(\Delta p_{it}^2, \Delta p_{is}^2) CV(\Delta p_{it}^2) CV(\Delta p_{is}^2)} \text{ for } t \neq s$$

where CV(.) denotes the coefficient of variation and corr(.,.) the correlation coefficient.

We use these equations to compute a measure of kurtosis robust to unobserved heterogeneity. In practice, we want to use information from several possible lags (the s's as different from t), rather than picking up a single particular lag s.

To compute the covariance terms in the expression above we as use an estimator of $E = E[(\Delta p_{it}^2)(\Delta p_{is}^2)]$ the following expression:

$$E = (1/\#Terms) \sum_{t,s \in T(i), t \neq s} (\Delta p_{it})^2 (\Delta p_{is})^2 \tag{42}$$

In practice, we consider the first K lags of squared price changes. So, the numerator of the formula (42) above is computed as:

$$S = 2 * [\sum_{t=2}^{T} (\Delta p_t)^2 (\Delta p_{t-1})^2 + \sum_{t=3}^{T} (\Delta p_t)^2 (\Delta p_{t-2})^2 + ... + \sum_{t=K+1}^{T} (\Delta p_t)^2 (\Delta p_{t-K})^2] \tag{43}$$

Denotig by NN the number of terms in equation (43), then $\#Terms = 2 * NN$, where:

$$NN = (T-1) + (T-2) + ... + (T-k) = T(T-1)/2 - (T-K-1) * (T-K)/2 \tag{44}$$

So when $K = T - 1$, $\#Terms = 2 * T(T-1)/2 = T(T-1)$ Then we recover

$$E = \frac{S}{T(T-1)}$$

Table B.10: Regression Results: Kurtosis Measurement - Constrained model

	PRODUCER PRICES			*CONSUMER PRICES*		
Identification	Cholesky	Cholesky	High-Freq. IV	Cholesky	Cholesky	High-Freq. IV
Long-run Restriction	Yes	No	Yes	Yes	No	Yes
Product FE	No	No	No	No	No	No
PANEL A: Outlier threshold - small price changes						
Kurt/Freq	0.184***	0.468**	0.166***	0.000529	0.0352	0.0150
	(0.0695)	(0.226)	(0.0615)	(0.0148)	(0.0399)	(0.0106)
Constant	-26.48***	-38.12***	-32.65***	-16.64***	-19.56***	-23.90***
	(4.153)	(14.05)	(3.506)	(1.915)	(5.654)	(1.555)
R^2	0.082	0.046	0.092	0.000	0.004	0.008
PANEL C: Outlier threshold - large price changes						
Kurt/Freq	0.0912*	0.233	0.0843*	0.00220	0.0259	0.0129
	(0.0499)	(0.146)	(0.0466)	(0.0113)	(0.0311)	(0.00812)
Constant	-25.07***	-34.59**	-31.51***	-16.87***	-19.47***	-24.08***
	(4.319)	(13.84)	(3.802)	(1.901)	(5.647)	(1.540)
R^2	0.063	0.036	0.074	0.000	0.003	0.010
Observations	118	118	118	223	223	223

Note: This table reports OLS results of the constrained model (equation (13)) for PPI products relating product-specific $CIR_T^{P_j}$ (expressed in %) to the ratio $Kurt/freq$ In Panel A, we have modified the thresholds defining very small price changes for the calculation of kurtosis: we have removed all price changes below 0.5% in absolute values (instead 0.1% in our baseline). In Panel B, we have modified thresholds defining very large price changes for the calculation of kurtosis (25% for instead of 15% in the baseline). Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table B.11: Regression Results: Kurtosis Measurement - Unconstrained model

	PRODUCER PRICES			*CONSUMER PRICES*		
Identification	Cholesky	Cholesky	High-Freq. IV	Cholesky	Cholesky	High-Freq. IV
Long-run Restriction	Yes	No	Yes	Yes	No	Yes
Product FE	No	No	No	No	No	No
PANEL A: Outlier threshold - small price changes						
$Freq/\bar{F}$	-7.146**	-21.84*	-6.046**	-7.279**	-23.86***	-5.020***
	(3.161)	(11.58)	(2.814)	(2.883)	(7.591)	(1.429)
$Kurt/\bar{K}$	6.486*	15.39	5.666*	4.166**	1.075	3.089***
	(3.813)	(11.96)	(3.207)	(1.644)	(3.720)	(1.103)
Constant	-18.29***	-12.51	-25.48***	-13.48***	6.255	-20.68***
	(4.610)	(15.25)	(3.660)	(3.734)	(8.946)	(2.125)
R^2	0.198	0.154	0.197	0.211	0.260	0.173
PANEL B: Outlier threshold - large price changes						
$Freq/\bar{F}$	-7.427**	-22.52*	-6.288**	-7.275**	-23.57***	-5.003***
	(3.185)	(11.64)	(2.868)	(2.806)	(7.527)	(1.398)
$Kurt/\bar{K}$	4.581	13.43	3.335	4.284**	2.538	3.138***
	(3.480)	(11.33)	(2.822)	(1.869)	(2.796)	(1.187)
Constant	-16.11***	-9.870	-22.91***	-13.63***	4.409	-20.79***
	(3.692)	(12.49)	(2.828)	(3.498)	(7.844)	(2.002)
R^2	0.197	0.157	0.191	0.218	0.258	0.178
Observations	118	118	118	223	223	223

Note: This table reports OLS results of the unconstrained model (equation (14)) relating product-specific $CIR_T^{P_j}$ (expressed in %) to the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$. In Panel A, we have modified the thresholds defining very small price changes for the calculation of kurtosis: we have removed all price changes below 0.5% in absolute values (instead 0.1% in our baseline). In Panel B, we have modified thresholds defining very large price changes for the calculation of kurtosis (25% for instead of 15% in the baseline). Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1